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The Effects of Ambient Shipping Noise on the Performance of Single and Multiple Channel Moment Detectors for Unknown Transient Signals

LISA A. PFLUG

Ocean Acoustics Branch
Acoustics Division

GEORGE E. IOUP
JULIETTE W. IOUP

University of New Orleans
Department of Physics
New Orleans, LA 70418

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Lisa A. Pflug, George E. Ioup*, and Juliette W. Ioup*

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*Department of Physics, University of New Orleans, New Orleans, LA 70418

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Second, third, and fourth order moments are used for passive detection of simulated transient signals embedded in both simulated independent Gaussian noise and measured noise. The measured noise was recorded by a vertical array located near the San Diego harbor and is dominated at low frequencies by ship-generated noise. The detectors do not assume noise or signal stationarity, and can use either one, two, or p channels of data for the *p-th* order moment detector. Detection simulations using six synthetic transient signals embedded in the measured noise reveal that the fourth order moment detector often detects better than the second order moment (or energy) detector in both the Gaussian noise and the measured noise. Although the application of one-dimensional passband filters before the moment calculations improves the performance of all three detectors, it induces additional detection gain for the higher order moment detectors.

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I. INTRODUCTION

Sonar targets have recently become quieter and their signatures more difficult to detect, especially in coastal and shallow-water areas. As a result, attempts are being made to exploit target-generated transient signals to improve detection. One approach is to capitalize on the nonlinear and nonGaussian nature of transient signals via the use of higher order statistics. Higher order statistical transient signal detectors have been shown to outperform the energy detector under certain conditions in simulations (Hinich 1990; Hinich and Wilson 1990; Ioup et al. 1990, 1993; Pflug et al. 1992b, 1994; Baugh et al. 1994; Walsh and Delaney 1995). In the most general sense, these detectors capitalize on the difference between the higher order statistic of the signal and that of the noise. For example, nonGaussian transient signals should be detectable in Gaussian noise since the higher order statistics will differ. Low-frequency measured noise, especially in a shallow-water area dominated by shipping activity, is expected to have moments that vary with time, i.e., to be nonstationary. The noise may not be Gaussian during the observation time used for detection, which is often of short duration. The noise is likely to be correlated between receivers and in time. The performance of the energy detector and higher order statistical detectors under these kinds of conditions is unknown and cannot readily be predicted with theoretical approaches.

The specific detection problem addressed in this report is that of passively detecting a single occurrence of a deterministic transient (energy) signal in noise. Theoretical predictions and computer simulations with Gaussian noise have shown that performance gain can be achieved with the third and fourth order moment detectors using information from p different hydrophones in a p-*th* order moment detector (Ioup et al. 1993; Pflug et al. 1992b, 1994, 1995a,b; 1997a,b). In this report, the signal is assumed to be received as an exact replica on either one, two, or p vertically spaced hydrophones, or channels (p is equal to the moment order), with channel data repeated in the higher order moments as needed. While the ambient shipping noise is somewhat coherent, it is not an exact replica from channel to channel. In reality, the signal will not necessarily be an exact replica on multiple channels either. However, this assumption is consistent with our detection requirement that the transient signal be more coherent than the noise across channels.

Transient signal detection is not simply related to power signal detection. With a power signal, averaging is used to enhance the likelihood of signal detection. However, since transient signals are generally nonstationary and often of short duration, averaging as done for detection of power signals may decrease rather than increase the likelihood of detection, especially for frequency-modulated and some broadband signals. Hence, our model does not include averaging. Detectors that do not use averaging do not depend on noise or signal stationarity to be successful and are, thus, applicable in a variety of situations (Pflug et al. 1992b, 1994, 1995b; Walsh and Delaney 1995), although short periods of relative stationarity of the noise are required for all detectors in practice to set an accurate threshold for a given false alarm rate. There is an additional advantage for third order statistical based detectors. It has been shown by Walsh and Delaney (1995) that for narrow-band signals, the third order moment, or bispectral, detector may not work well if averaging is used since the terms containing signal times noise on average are theoretically zero. In contrast, if the raw estimate, or single-frame, bispectrum is used, these terms contribute to the signal-present (*s-p*) detection statistic but not the signal-absent (*s-a*) detection statistic, leading to improved detection performance. Since the trispectrum consists of two distinct principal domains, one of which behaves like the principal domain of the bispectrum, it too can benefit from using the raw estimate rather than an average estimate (Pflug et al. 1992a). The drawback when using detectors that do not require stationarity is, of course, the loss of signal gain and/or noise suppression that could be obtained through averaging.

In Sec. II, moment detectors are defined, and the detection algorithm is presented. In Sec. III, a discussion of previous research on the stationarity and Gaussianity of ambient noise is given, and the ambient shipping noise data used in this study are discussed in Sec. IV. The set of test signals employed to evaluate detector performance in simulations is introduced in Sec. V. The results of computer simulations showing the performance of the moment detectors for signals embedded in simulated Gaussian noise and in ambient shipping noise are given in Sec. VI. This section also includes a summary of the detection results for ease of comparison. Finally, in Sec. VII, conclusions are drawn from the analysis.

II. MOMENT DETECTORS AND ALGORITHM DESIGN

The *p-th* order moment detector uses the zero-lag value of the *p-th* order moment, or correlation, as a test statistic and compares it to a predetermined threshold. A decision is then made about whether the test data contains a signal embedded in noise, or only noise. The zero-lag value of a *p-th* order moment is defined by

$$m_p^x = \sum_{k=0}^{N-1} x^p(t)\Delta t\,, \tag{1}$$

for a single channel of time-domain data, $x(t)$, with $t = k\Delta t$. For two distinct channels of data (denoted by subscripts on $x(t)$), the second through fourth order moments are defined by

$$m_2^x = \sum_{k=0}^{N-1} x_1(t)x_2(t)\Delta t \tag{2a}$$

$$m_3^x = \sum_{k=0}^{N-1} x_1^2(t)x_2(t)\Delta t \tag{2b}$$

$$m_{4_{(2,2)}}^x = \sum_{k=0}^{N-1} x_1^2(t)x_2^2(t)\Delta t \tag{2c}$$

$$m_{4_{(3,1)}}^x = \sum_{k=0}^{N-1} x_1^3(t)x_2(t)\Delta t\,. \tag{2d}$$

Note that the fourth order moment can be defined in two ways as shown in Eqs. (2c) and (2d). The subscripts (2,2) and (3,1) indicate the powers on the two data channels. When the number of channels used in the moments is equal to the moment order (the p channel case), the third and fourth order moments are given by the equations

$$m_3^x = \sum_{k=0}^{N-1} x_1(t)x_2(t)x_3(t)\Delta t \tag{3a}$$

$$m_4^x = \sum_{k=0}^{N-1} x_1(t)x_2(t)x_3(t)x_4(t)\Delta t\,. \tag{3b}$$

For second order, the two channel case, Eq. (2a), is the p channel case. When a signal is present, $x_i(t) = s(t) + n_i(t)$, where $s(t)$ is a transient source sequence and $n_i(t)$ is a noise sequence received on one channel. When the signal is absent, $x_i(t) = n_i(t)$.

Another interpretation of the *p-th* order moment is that it represents the area, volume, or hypervolume beneath the corresponding moment spectrum. In this sense, higher order moments are higher order extensions of the energy. Hence, the second, third, and fourth order moments of a single channel of data, $x(t)$, are equivalent to the area, volume, and hypervolume beneath the energy spectrum (ES), the energy bispectrum (BS), and the energy trispectrum (TS), respectively

$$m_2^x = \sum_{j=-N/2}^{N/2} ES\,\Delta f = \sum_{j=-N/2}^{N/2} X(f)X^*(f)\,\Delta f = \sum_{j=-N/2}^{N/2} |X(f)|^2\Delta f \tag{4a}$$

$$m_3^x = \sum_{j_1=-N/2}^{N/2}\ \sum_{j_2=-N/2}^{N/2} BS\,\Delta f_1\Delta f_2 = \sum_{j_1=-N/2}^{N/2}\ \sum_{j_2=-N/2}^{N/2} X(f_1)X(f_2)X^*(f_1+f_2)\Delta f_1\Delta f_2 \tag{4b}$$

$$m_4^x = \sum_{j_1=-N/2}^{N/2}\ \sum_{j_2=-N/2}^{N/2}\ \sum_{j_3=-N/2}^{N/2} TS\,\Delta f_1\Delta f_2\Delta f_3\,, \tag{4c}$$

$$= \sum_{j_1=-N/2}^{N/2}\ \sum_{j_2=-N/2}^{N/2}\ \sum_{j_3=-N/2}^{N/2} X(f_1)X(f_2)X(f_3)X^*(f_1+f_2+f_3)\Delta f_1\Delta f_2\Delta f_3\,,$$

where $X(f)$ is the Fourier transform of $x(t)$ and $f = j\Delta f$. When two or more channels of data are used in the moment calculation, the ES, BS, and TS consist of products of $X_1(f)$, $X_2(f)$, $X_3(f)$, and $X_4(f)$, as appropriate. Although the BS and TS in Eqs. (4b) and (4c) are complex functions, the imaginary parts are odd; hence, the summations and moments are real.

In nonstationary noise, the noise moments change with time and the detection of the signal at different times in the noise might also change. One way to study detection performance is to place the signal in each of a sequence of processing windows and calculate the moments. This results in a probability of detection (P_d) in the sense that the detectors can be said to register a true detection for a certain fraction of the windows. This is a modification of the P_d as defined for a traditional Receiver Operating Characteristic (ROC) curve, which assumes that an ensemble of stationary noise realizations is available for testing. The probability of false alarm (P_{fa}) for nonstationary noise can be defined in a similar manner. This approach is used here for the measured noise with the understanding that the results are a function of the time segment of the nonstationary noise chosen for analysis.

An additional processing step may be added to the detection algorithm. Previous simulations of moment detector performance in Gaussian noise have shown that the performance of the third and fourth order moment detectors can improve relative to the second order moment detector through the use of simple passband filtering (Ioup et al. 1993; Pflug et al. 1994). That is, one-dimensional passband filtering of the received channel data prior to applying the detector has a nonlinear and advantageous effect for the higher order moments relative to the second order. This process is referred to as prefiltering. In passive detection, it is generally unlikely that a good estimate of the signal passband will be available unless one is searching for a specific class of signals. However, prefiltering can be used by dividing the data passband into appropriate sub-bands and applying the moment detectors successively to each sub-band. The use of prefiltering in its simplest form, as done in this report, can determine whether or not it has merit for passive sonar applications.

III. AMBIENT SHIPPING NOISE

Moment detectors essentially capitalize on the difference between the moments of the signal and the moments of the noise in which the signal resides. Detection performance is also affected by interaction between the signal and noise, which is assumed to be small for finite-length sequences. If p channels of data are used in the moment definitions, then the *s-a* and *s-p* probability density functions (PDF), and consequently detection performance, depend only on the first two moments of the noise. If channels are repeated so that two channels are used in the *p-th* order moment, then detector performance depends on moments of the noise of orders p_1 and p_2, where $p = p_1 + p_2$, and p_1 and p_2 represent the number of times each of the two channels is repeated in the correlation. If a single channel of data is used, then the *p-th* order moment detector depends on the *p-th* order moment of the noise. A derivation of these results is given in App. A.

Ocean noise is often assumed to be Gaussian distributed, which naturally suggests using higher order moments to detect nonGaussian signals. However, in some situations, the ocean noise may be nonGaussian, especially if it is dominated by one or more nearby ships. It is well accepted that shipping is the dominant noise source at frequencies between 20 and 500 Hz and that there are time-dependent fluctuations in the noise that may induce nonstationarity (Wenz 1962). The timescale of fluctuations depends on the number and density of ships, the proximity of shipping traffic to the receiver, the receiver location, and propagation characteristics of the area.

The literature shows that deep-water ambient noise due to shipping can be both nonstationary and nonGaussian. One of the earlier works in the study of noise stationarity and Gaussianity concludes that ambient noise at frequencies less than 2500 Hz recorded off the southern California coast, in the Bering Straits, and in the North Pacific is generally Gaussian when it is essentially free from contamination, such as nearby shipping, biologics, etc. (Calderon 1964). Calderon also concludes that ambient shipping noise in the San Diego harbor is nonGaussian over a 7-min time period at 20–1200 Hz. This conclusion is based on the assumption that the noise is stationary over the 7-min time period. In a study by Arase and Arase (1968), noise recorded in deep water near Bermuda is shown to be stationary for less than 3 min over a frequency band of 100–1600 Hz. They also conclude that more than half of the noise samples in time intervals ranging from 10–40 s are Gaussian, with stronger nonGaussianity in longer time periods. Jobst and Adams (1977) show that distant deep-water shipping noise recorded in the North Atlantic appears stationary for less than 22.9 min at 75 Hz and 8.2 min at 260 Hz. Their analysis is restricted to noise without extremely narrow-band spectral energy, which is expected in the presence of nearby shipping noise. Moreover, the time periods of stationarity are likely to be much shorter in the presence of nearby shipping, especially for shallow-water areas with highly variable bathymetry and geoacoustics.

Recently, research has been published presenting new ways of looking at nonGaussianity. Using a third order Gaussianity test by Hinich (1982), Brockett et al. (1987) have shown that ambient ocean noise appears Gaussian for periods over a minute, but nonGaussian for shorter periods, while local shipping noise (from one nearby ship) is nonGaussian for both long and short time periods. Hinich et al. (1989) also show that the towing platform in an experiment has strong bispectral components. Richardson and Hodgkiss (1994) use a normalized third order statistic to determine that a 45-s duration deep-water noise sequence is nonGaussian. In one of the few applications of fourth order statistics, Dalle Molle and Hinich (1995) use the trispectrum to conclude that a 6-min time series generated by two ships at distances of 460 and 635 m from a sonobuoy in the eastern Atlantic Ocean appears to be nonGaussian for eight out of twelve 0.5-min time segments. In contrast, a second 4-min time series during which the two ships are 2000 and 5300 m from the sonobuoy appears similar to the Gaussian ambient sea noise.

The research to date in the areas of stationarity and Gaussianity of ambient shipping noise indicates that these assumptions must be used cautiously for real-world applications, especially in shallow water. This, as well as the nonstationary nature of many transient signals, motivates the development of detectors that do not require stationarity for application.

IV. SHIPPING NOISE NEAR THE SAN DIEGO PORT

The ambient shipping noise used for evaluation of the moment detectors was taken during the Shallow Water Evaluation Cells Exercise-3 (SWellEX-3) that occurred during Jul–Aug 1994 near the port of San Diego, CA (Bachman et al. 1996). Ambient noise measurements were recorded on a vertical 64-element array with 2-m spacing, located in water approximately 200 m deep, with the bottom hydrophone (channel 1) approximately 3.4 m above the seafloor. The area is characterized by heavy shipping traffic, including military, commercial, and recreational vessels. The noise is sampled at 1500 samples/s. As part of the calibration, mooring platform self-noise in the data is reduced or removed and a high-pass Butterworth filter of order nine with a cutoff frequency of 15 Hz is applied to reduce the effects of sensor motion, or flow noise, that appears in the uppermost phones.

Simultaneous 3-min segments of noise from four hydrophones or channels (channels 2, 16, 31, and 43) are used in the detection analysis. Local shipping traffic was recorded by radar during the experiment and is depicted in Fig. 1 for the 3-min observation time (Holliday 1995). The shipping traffic is quite heavy and there are several ships in close proximity to the array. The numerous smaller recreational craft that frequent the area were not reliably tracked by the radar system during the experiment.

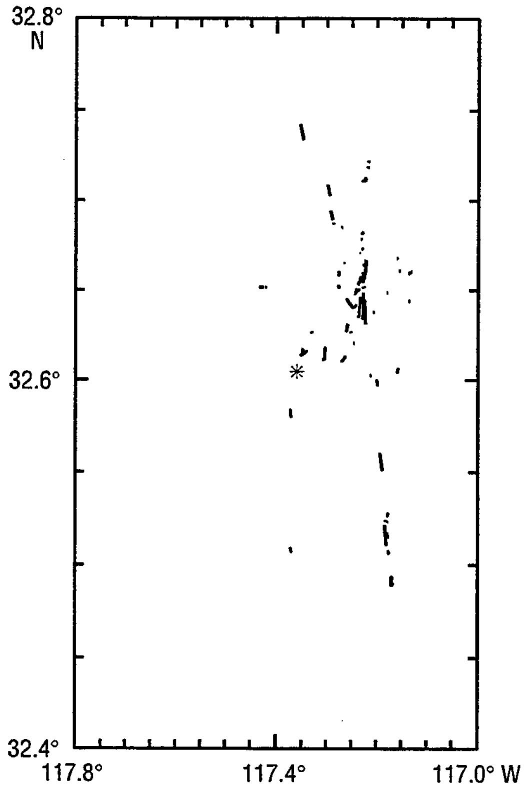

Fig. 1 — The SWellEX-3 shipping traffic during the 3-min segment and the array position (denoted by an asterisk)

The 3-min segment of calibrated noise from channel 2 and its Fourier transform are shown in Fig. 2. The same is shown for channel 43 in Fig. 3. Since the Fourier transform includes the entire 3 min of noise, it does not show any nonstationarities in the data. However, it does show the overall frequency distribution of the shipping noise.

To test the 3-min segment of noise for Gaussianity, the one-sample Kolmogorov-Smirnov (K-S) test for goodness of fit is used. The K-S statistic, D_n, is defined as the maximum difference between the theoretical Gaussian cumulative distribution, $G(x)$, and the experimental cumulative distribution $S_N(x)$, i.e.

$$D_N = \max|G(x) - S_N(x)|\,, \tag{5}$$

where N is the number of sample points. D_N is then compared to a value determined by the confidence level, which is derived according to

$$D_N < \lambda / \sqrt{N}\,, \tag{6}$$

with λ a function of the confidence level. Based on this comparison, the assumption of Gaussianity is either accepted or rejected. The K-S test is applied to test for local Gaussianity by sliding a 1-s window over the shipping noise with 99% (1485 data points) of overlap. This overlap is chosen to avoid aliasing of the second and higher order moments. The results of the K-S test for channels 2, 16, 31, and 43 are shown in Fig. 4. This figure shows the K-S statistic of Eq. (5) for each processing window over the 3 min of data. At a 95% confidence level, denoted by the horizontal

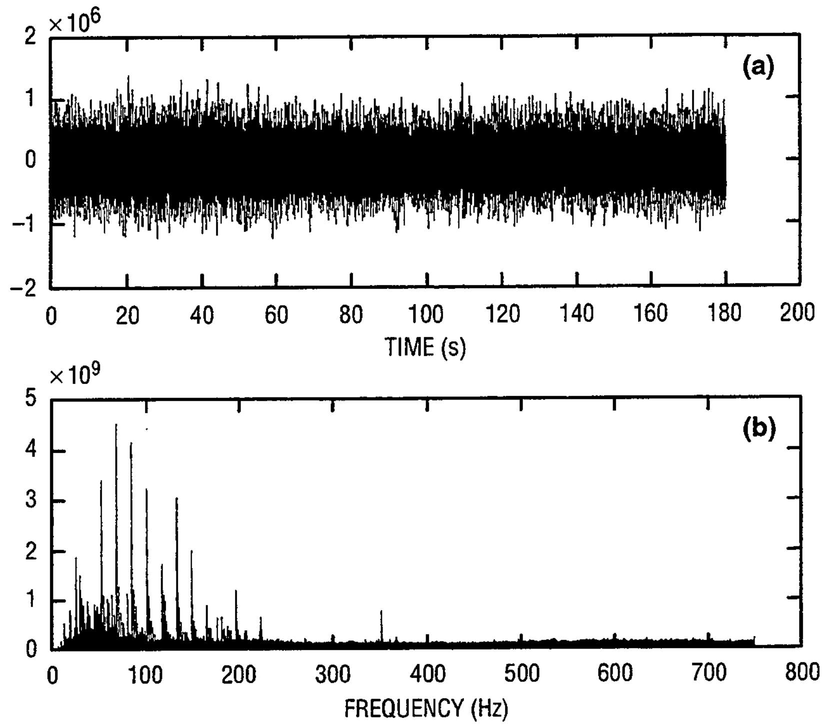

Fig. 2 — (a) The SWellEX-3 noise received at phone 2 and (b) the magnitude of its Fourier transform

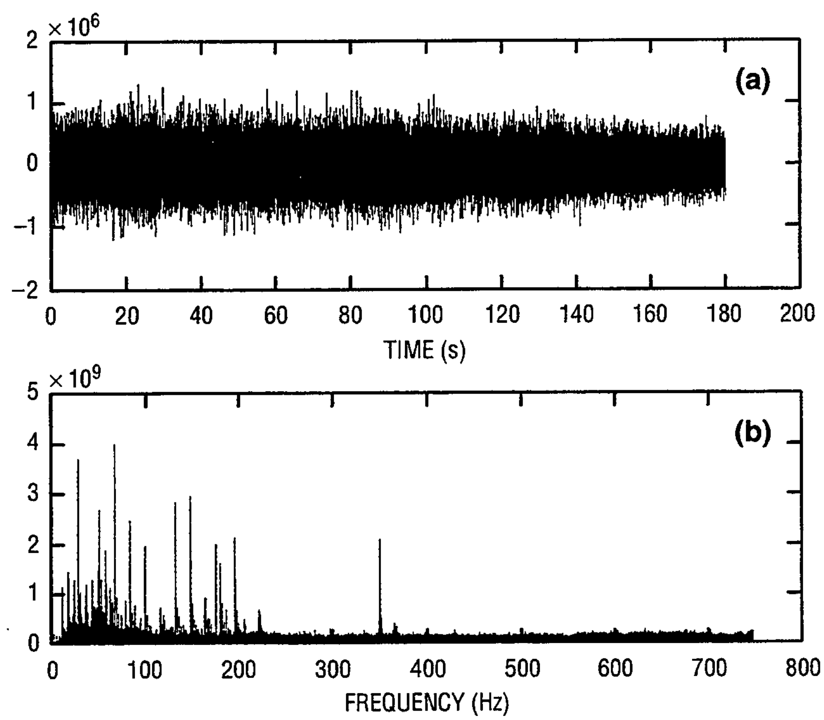

Fig. 3 — (a) The SWellEX-3 noise received at phone 43 and (b) the magnitude of its Fourier transform

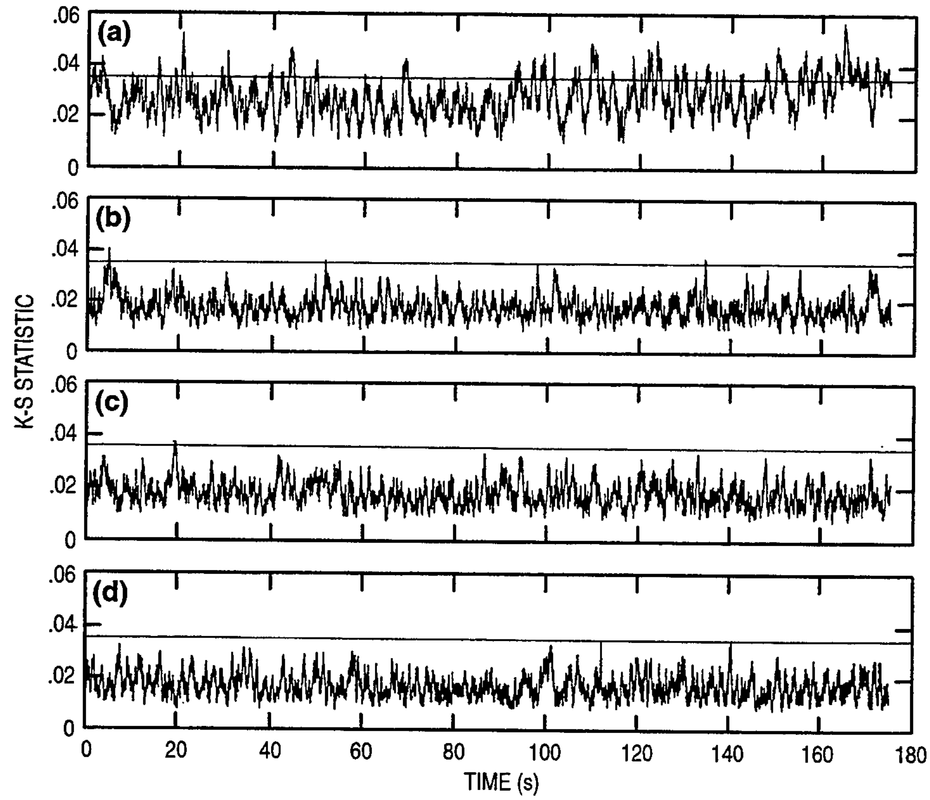

Fig 4 — Results of the K-S test for the shipping noise received at channels (a) 2, (b) 16, (c) 31, and (d) 43

Table 1 — Percentage of NonGaussian Processing Windows for Four Channels of SWellEX-3 Noise

CHANNEL NUMBER	% NONGAUSSIAN
2	18.39
16	0.2000
31	0.0575
43	0.0058

lines in Fig. 4, the noise at channel 2 differs from a Gaussian distribution for a significant percentage of the processing windows, but the others do not (see Table 1).

A 27-min segment of noise (containing the 3-min segment discussed here) is analyzed for stationarity. Stationarity is evaluated using a runs test with a two-sample K-S test discriminant (the two-sample K-S test replaces the theoretical distribution in the one-sample K-S test with a second experimental distribution). The results show that the noise contains random periods of stationarity ranging from 0.1 s to over 9.1 min. A detailed study of the stationarity and Gaussianity of the SWellEX-3 noise is included in Pflug et al. (1997c).

V. TEST SIGNALS

A set of six different simulated transient signals ranging from narrowband to broadband is used to evaluate the performance of the moment detectors (Fig. 5). The signals are:

(1) a simulated 20-Hz finback *whale transient* that has piece-wise linear amplitude modulation and nonlinear frequency modulation;

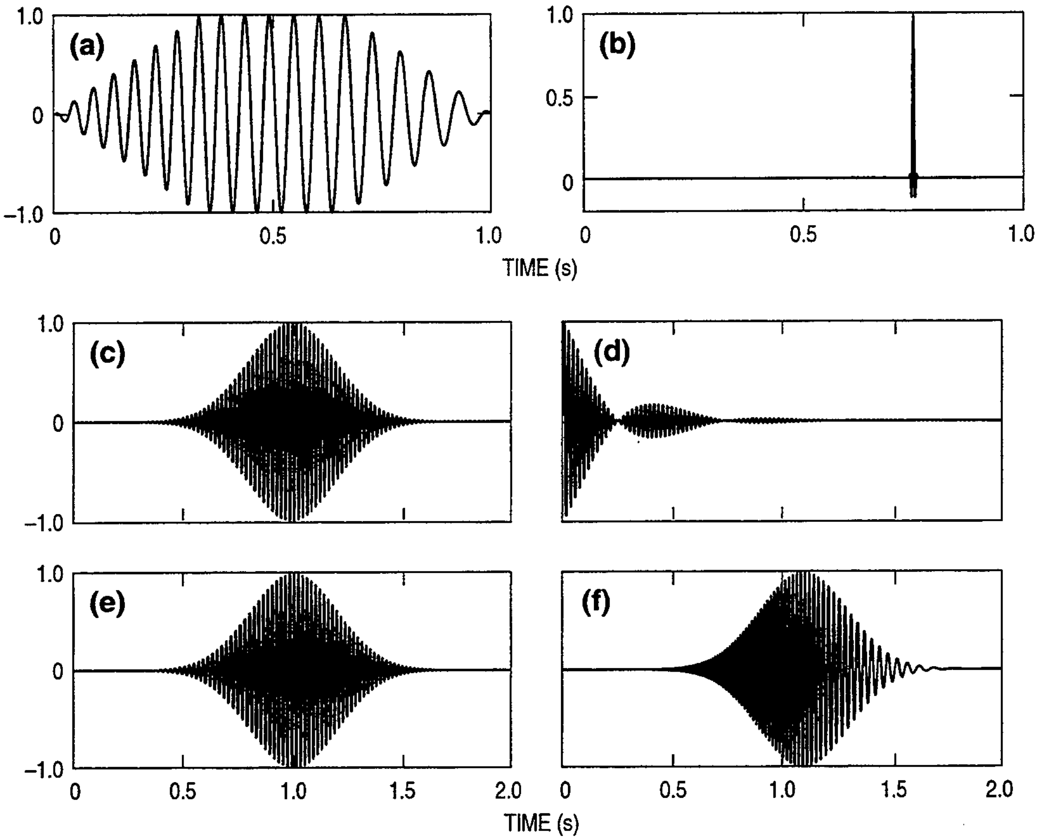

Fig. 5 — The six test transient signals, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep

(2) a *narrow pulse* that has a flat magnitude spectrum to approximately 80 Hz and a smooth rolloff to 256 Hz;

(3) a *50-Hz sinusoid*, amplitude modulated by a Gaussian;

(4) a *49- to 51-Hz sinusoid* that contains two beating sinusoids, one at 49 Hz and one at 51 Hz, amplitude modulated by a Gaussian;

(5) a *linear FM sweep* from 41–59 Hz, amplitude modulated by a Gaussian; and

(6) a *nonlinear FM sweep* from 120–0 Hz, amplitude modulated by a Gaussian.

Henceforth, the signals will be referred to by the words in italics. The Fourier magnitude spectra of the signals are given in Fig. 6. For the whale and narrow pulse signals, the processing window durations are defined to be 1 s long, and for the last four signals, the durations are defined to be 2 s long.

It has been shown that even though the nonnormalized moments, as defined by Eq. (1), are used as the test statistic in the detectors, normalized moments are more directly related to the relative performance of the second, third, and fourth order moment detectors, at least for stationary Gaussian

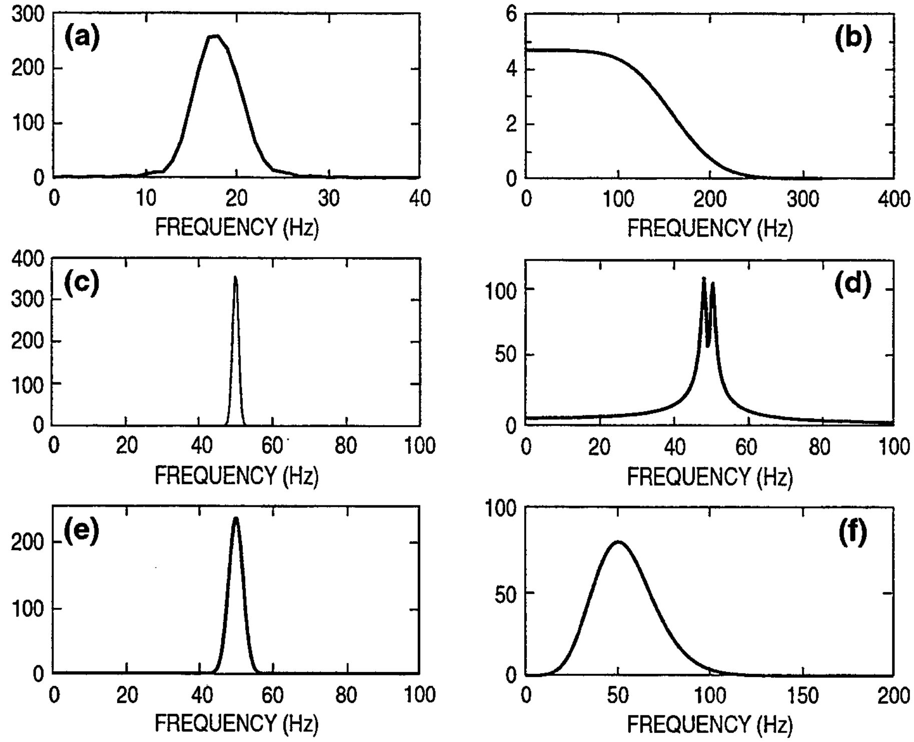

Fig. 6 — The Fourier transform magnitude of the six test transients, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep

noise (Pflug et al. 1992b). The normalized moments (variance, skewness, and kurtosis) include a division by appropriate powers of the variance. If the mean is defined by

$$\bar{x} = \frac{1}{N} \sum_{k=0}^{N-1} x(t) , \tag{7}$$

then the variance, skewness, and kurtosis are defined by

$$\sigma_x^2 = \frac{1}{N} \sum_{k=0}^{N-1} \left[s(t) - \bar{x} \right]^2 \tag{8a}$$

$$S_x = \frac{1}{N} \sum_{k=0}^{N-1} \left[\frac{x(t) - \bar{x}}{\sigma_x} \right]^3 \tag{8b}$$

$$K_x = \left\{ \frac{1}{N} \sum_{k=0}^{N-1} \left[\frac{x(t) - \bar{x}}{\sigma_x} \right]^4 \right\} - 3 . \tag{8c}$$

The subtraction of 3 in the expression for kurtosis is an adjustment to shift the kurtosis of Gaussian distributed data to zero. The skewness for such data is already zero. The signal variance,

Table 2 — Normalized Moments for the Six Test Signals

SIGNAL	MEAN	VARIANCE	SKEWNESS	KURTOSIS
Whale	-9.13×10^{-5}	0.278	7.82×10^{-3}	-0.731
Narrow Pulse	3.12×10^{-3}	2.67×10^{-3}	15.7	263.0
50-Hz Sinusoid	2.52×10^{-8}	8.63×10^{-2}	1.27×10^{-6}	3.14
49- to 51-Hz Sinusoid	1.58×10^{-3}	2.05×10^{-2}	0.343	18.31
Linear FM Sweep	3.14×10^{-8}	8.63×10^{-2}	-5.56×10^{-7}	3.14
Nonlinear FM Sweep	-2.95×10^{-6}	8.64×10^{-2}	3.08×10^{-5}	3.14

and thus the skewness and kurtosis, affect the performance of the moment detectors through the signal-to-noise ratio (SNR) (Pflug et al. 1995 a,b), which is defined by

$$SNR = \frac{\sigma_s}{\sigma_n}, \tag{9}$$

and converted to power in decibels by taking 20log (SNR).

For future reference, the normalized moments of the six test signals within the defined processing window durations are given in Table 2. As indicated by the values in the table, all of the signals have negligibly small means. The skewness and kurtosis values indicate that the narrow pulse signal, and to a lesser degree the 49- to 51-Hz sinusoid signal, have significantly nonGaussian distributions.

VI. COMPUTER SIMULATIONS

Each set of measured noise is nonstationary, requiring a deterministic, time-dependent approach for the detection performance calculations. The chosen technique reflects an actual nonstationary noise detection scenario for which the detection threshold would be continuously updated over time to achieve a selected false alarm rate. This method was tested with synthetically generated stationary noise for the same sample size as the measured data. The small number of processing windows used in the calculations resulted in up to a 3.5 dB variation in SNR at $P_d = 0.5$ (standard deviation $\approx$0.7 dB). Therefore, a similar level of variation in the performance from one noise set to the next should be expected for the measured noise from one 3-min segment to another, even when the noise is stationary. For the stationary simulated noise, we lengthen the noise sets to 1000 processing window lengths to reduce the variability in the calculated SNRs.

The test signals are used in computer simulations with uncorrelated Gaussian noise to study the performance of the second, third, and fourth order moment detectors using one, two, and p channels of data. Since the measured noise is nonstationary, averaging to obtain predictive detection statistics is not possible, and the number of processing windows used to derive the detection results is based on the 3-min noise segment. For the two 1-s signals (the whale and pulse signals), a 1-s processing window is used with no overlap. The number of processing windows in the 3-min measured noise segment is 180, which permits detection at a minimum nonzero P_{fa} value of 0.0056 (smaller values of P_{fa} would require extrapolation). For the remaining 2-s signals with a 2-s processing window and no overlap, the number of processing windows is 90 and the minimum nonzero P_{fa} permitted is 0.011. All subsequent detection results, including those for the Gaussian noise, assume these minimum P_{fa} values. In Pflug et al. (1995 a,b), it is shown that in independently and identically distributed (i.i.d.) noise, passive higher order moment detectors improve in performance relative to the second order moment detector as the P_{fa} is decreased, or the processing window size or sampling interval is increased. It is as yet unknown how well these trends extend to the measured noise, which is correlated from channel to channel and over time.

The detection algorithm begins with a calculation of the average standard deviation of the noise in each of the processing windows included in the simulation. A local average must be used if the noise is nonstationary because the standard deviation of the noise may vary with time. The modified amplitude SNR is defined by $SNR = \sigma_s/\bar{\sigma}_n$, the ratio of the standard deviation of the signal to the average of the standard deviations of the noise over both time and channels. The average noise standard deviation is used to determine the appropriate signal standard deviation or level required to achieve a desired SNR, and the signal is amplitude adjusted accordingly. When one channel is used for detection, the average noise standard deviation calculation is straightforward. When two channels are used, the average is simply the average over the processing windows in both channels. However, since the noise is assumed to be nonstationary, when p channels of data are used in the simulations, one of two routes can be chosen: (1) the same signal level can be used for one SNR, based on one average noise standard deviation over the maximum number of channels included in the simulations (which is four for the fourth order moment) or (2) the signal level can be different for the second, third, and fourth order moment calculations, based on the individual average noise standard deviations over two, three, and four channels, respectively. In this work, the first option is used since it permits fairer comparison between detectors by using a constant signal amplitude across channels for a given SNR. The second order moment curves in the following simulations with two and p channels of the shipping noise may not be exactly the same as a result of this choice. For stationary noise, the SNR is as given in Eq. (9) since the average is not changing with time.

Next, the second through fourth order moments of the noise in each processing window are calculated to obtain the *s-a* moments. The amplitude-adjusted signal is added to each processing window and then the corresponding *s-p* moments are calculated for each detector. A threshold based on the chosen P_{fa} is determined from the *s-a* moments and then used to determine the corresponding P_d from the *s-p* moments for each detector.

The third order moments of the signals are positive, except for the linear FM Sweep (see Table 2). After prefiltering, however, the whale, 50-Hz sinusoid, and 49- to 51-Hz sinusoid have negative third order moments, while the pulse, linear FM sweep, and nonlinear FM sweep have positive third order moments. The detection algorithm used in this study assumes that the third order moment is always positive. When passively detecting a transient in practice, it will not be known whether the third order moment of the signal is positive or negative, and a simultaneous positive and negative peak detection algorithm could be implemented. An additional problem with using the third order moment as a detection statistic lies in the fact that the maximum positive or negative peak of the third order correlation does not always exist at the zero correlation lag, and the value at zero lag may be small. These problems can lead to unexpected results when using third order moments as defined in Eqs. (2b) and (3a) for detection.

A. Detection Results in Gaussian Noise

The moment detectors are first evaluated with the six test signals in Gaussian noise. In the first set of simulations, only one channel of data is assumed to be available for processing. The pictorial results for the six test signals are shown in Fig. 7. The plots show P_d vs. SNR in decibels. The results indicate that for all but the narrow pulse, the second order moment detector is slightly better than or the same as the fourth order moment detector, and the third order moment detector performs poorly. For the narrow pulse signal, the fourth order moment detector performs best, followed by the third and then second order moment detectors. This result is predictable from the normalized moment values given in Table 2, which show the narrow pulse signal to be much more nonGaussian than the other signals.

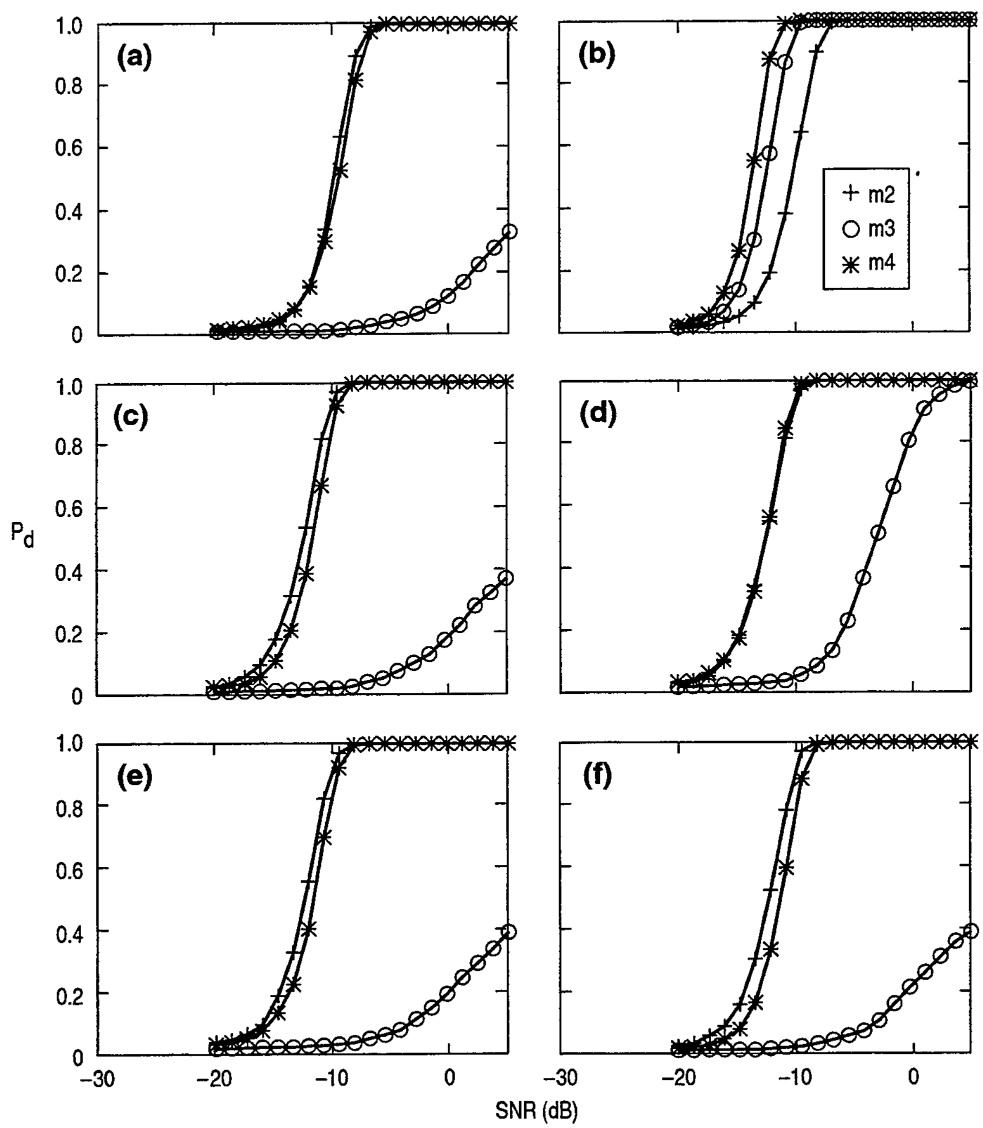

Fig. 7 — Single-channel detection simulations for the test signals in Gaussian noise, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep

In the second set of simulations, the moment detectors use received data from two channels, with both $m^x_{4(2,2)}$ and $m^x_{4(3,1)}$ used for the fourth order moment. The results with $m^x_{4(2,2)}$ are shown in Fig. 8. The addition of a second channel of data appears to improve the performance of the second and fourth order moment detectors by about 1–2 dB in most cases with the second order moment detecting slightly better than the fourth for the whale, 50-Hz sinusoid, linear FM sweep, and the nonlinear FM sweep. As in the single-channel case, both the third and fourth order moment detectors surpass the second for the narrow pulse. Virtually no difference is seen in performance when $m^x_{4(3,1)}$ is instead used for the fourth order moment, as shown in Fig. 9.

Finally, p channels of data are used in the p-*th* order moment detector with the results shown in Fig. 10. Thus, the third and fourth order detectors in this case use more channel information than

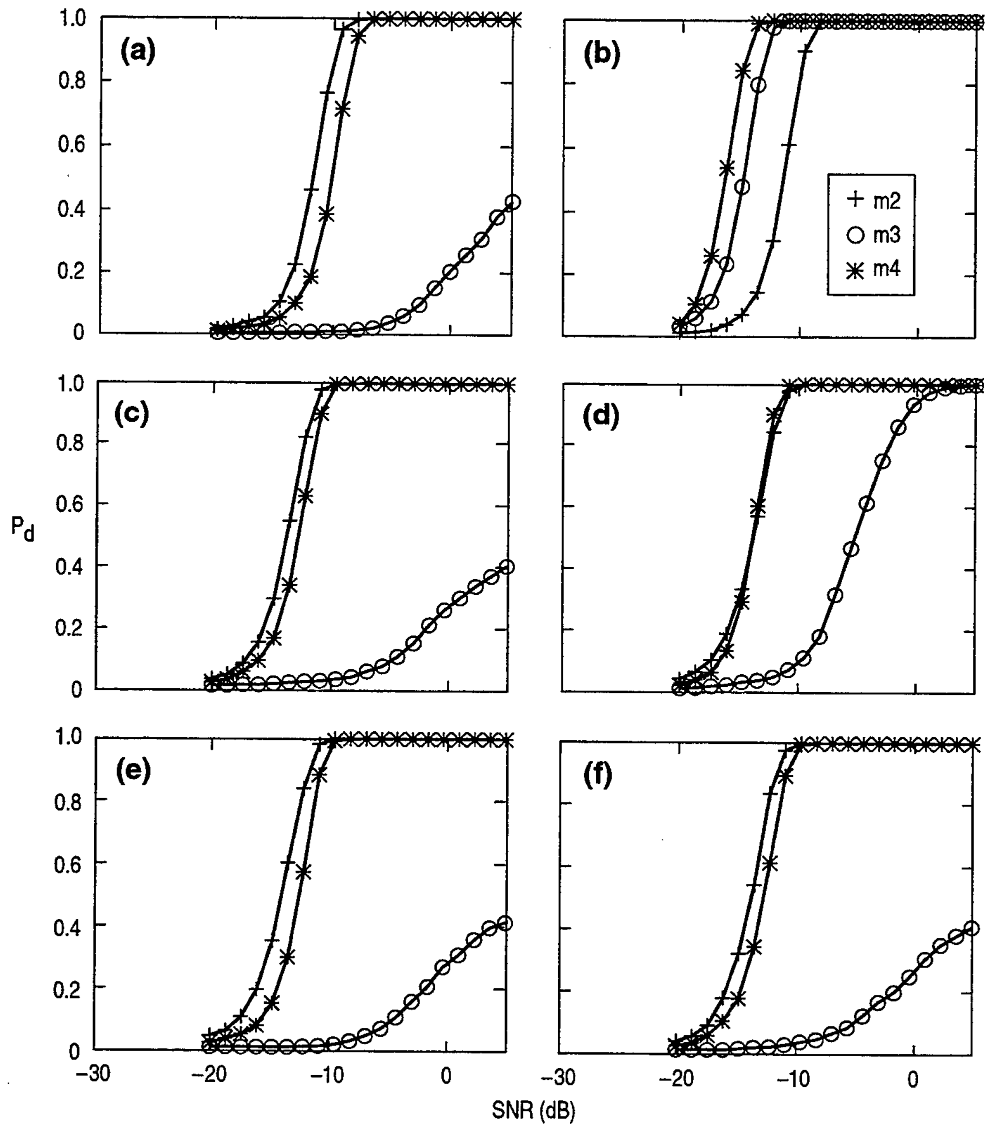

Fig. 8 — Two-channel detection simulations for the test signals in Gaussian noise. The fourth order moment detector is $m^x_{4(2,2)}$, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep.

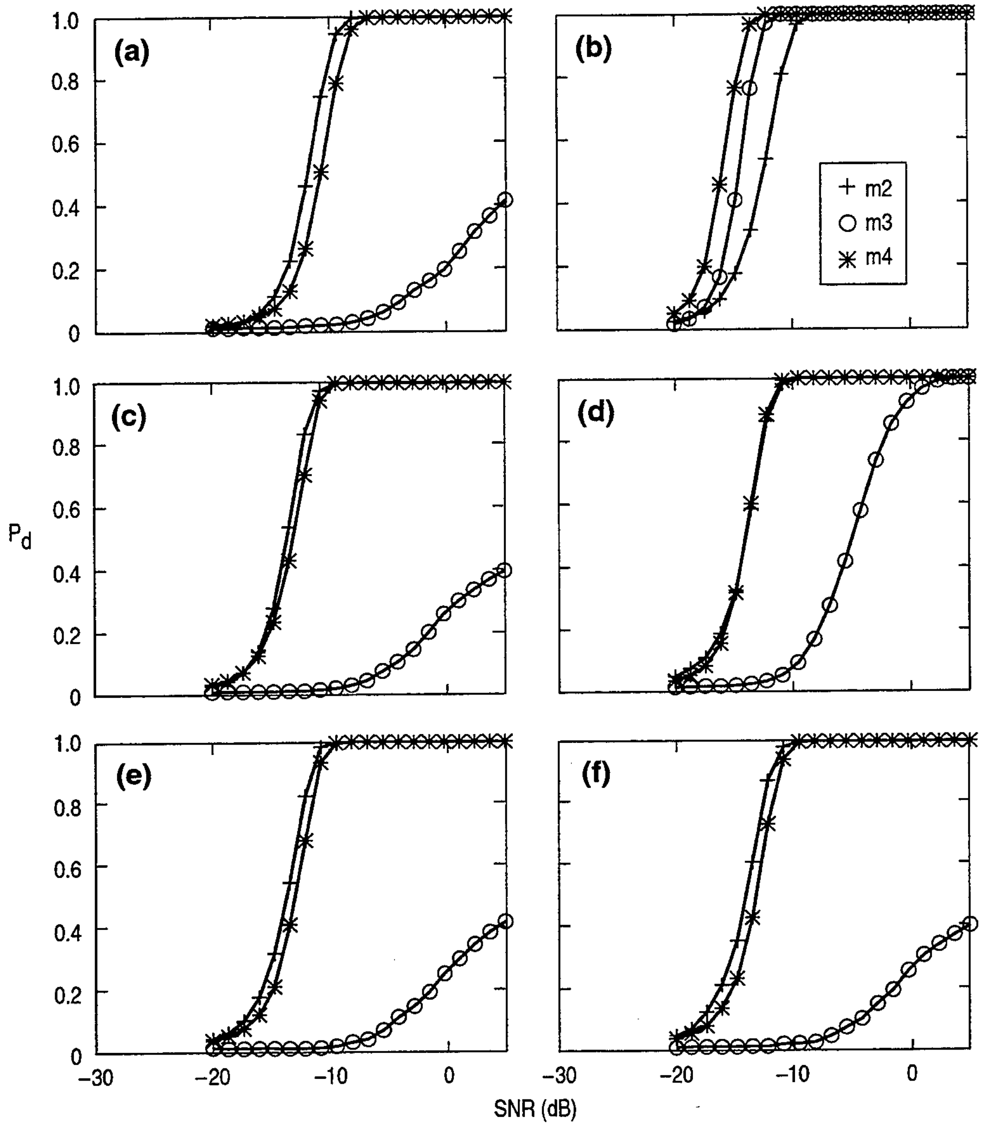

Fig. 9 — Two-channel detection simulations for the test signals in Gaussian noise. The fourth order moment detector is $m^{x}_{4(3,1)}$, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep.

the second order detector. The second order moment detector could also be used with four channels of data by applying it separately to two pairs of channels and subsequently correlating the result. Such an application would require more multiplications than a fourth order detector, and thus would require more computational time. An exploration of this technique is not included in this report. For the p channel case, the performance of the third order moment detector is still poor for all but the narrow pulse signal. The performance of the fourth order moment detector improves for the narrow pulse from a SNR gain over the cross correlation detector at $P_d = 0.5$ of 4.96 dB for the $m^{x}_{4(2,2)}$ detector to 5.97 dB for the p channel case. For the third order moment detector and the narrow pulse, the corresponding improvement is 3.37 to 4.00 dB. The performance of the fourth order moment detector remains the same for the 49- to 51-Hz sinusoid and declines for the whale signal, 50-Hz sinusoid, linear FM sweep, and nonlinear FM sweep.

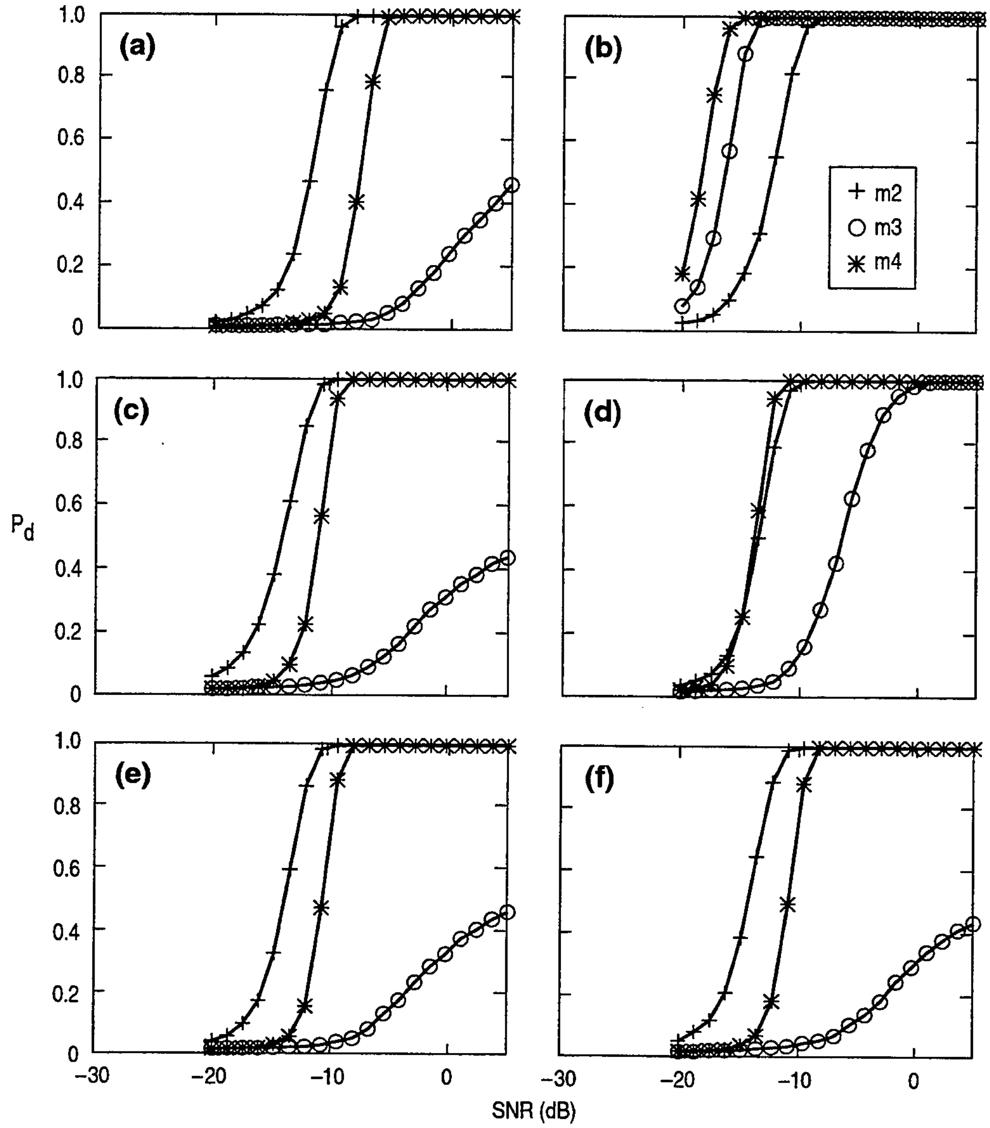

Fig. 10 — *p* channel detection simulations for the test signals in Gaussian noise, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep

B. Detection Results in Ambient Shipping Noise

To compare to the uncorrelated Gaussian noise simulation results using a single channel of data, simulations are done using a single channel of ambient shipping noise recorded during the SWellEX-3 experiment. The results using only channel 2 are shown in Fig. 11 and the results using only channel 43 are shown in Fig. 12.

As expected, for all signals in the channel 2 noise, detection performance is degraded in the ambient shipping noise compared to performance in Gaussian noise. However, the relative performance of the three detectors is different. In general, the performance of the third order moment detector is relatively better in the ambient shipping noise than Gaussian noise for all signals, but

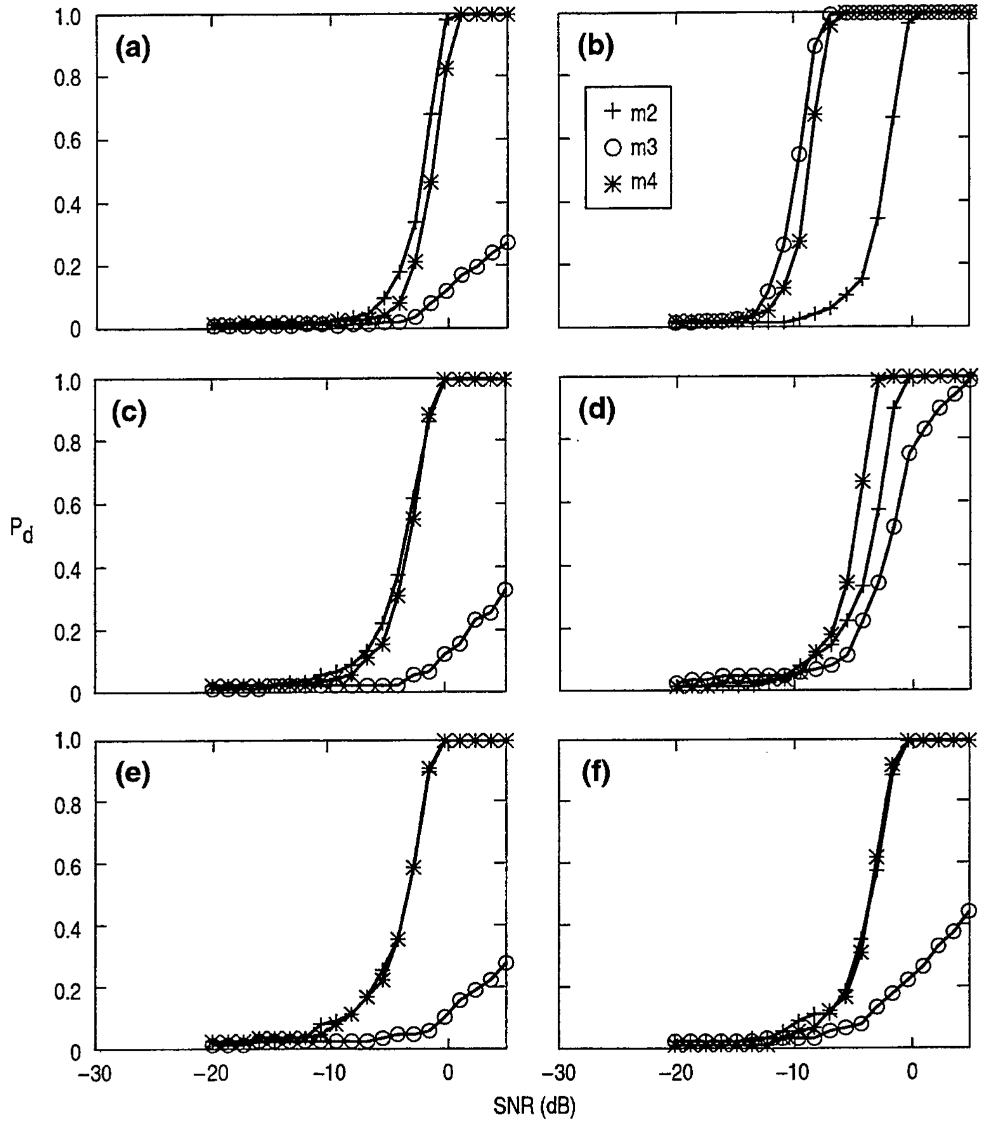

Fig. 11 — Single-channel detection simulations for the test signals in the SWellEX-3 noise, channel 2, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep

is still quite poor compared to the second and fourth order moment detectors for all but the narrow pulse and 49- to 51-Hz sinusoid signals. For the narrow pulse in the ambient shipping noise, the third order moment detector surpasses the second and fourth order moment detectors. Gains for the third and fourth order moment detectors over the second order moment detector at $P_d = 0.5$ are 7.44 dB and 6.47 dB, respectively. For the 49- to 51-Hz sinusoid, the fourth order moment detector performs better than the second order moment detector in the ambient shipping noise by about 1.58 dB at $P_d = 0.5$, but does not perform better in the Gaussian noise. The third order moment detector for this signal also shows significant gains relative to the other detectors in the ambient shipping noise over the Gaussian noise. Whereas the second order moment detector performs better than the fourth order moment detector in Gaussian noise by about 0.75 dB for the 50-Hz sinusoid and the linear FM sweep and by 0.97 dB for the nonlinear FM sweep, they are essentially equivalent when the ambient shipping noise is used.

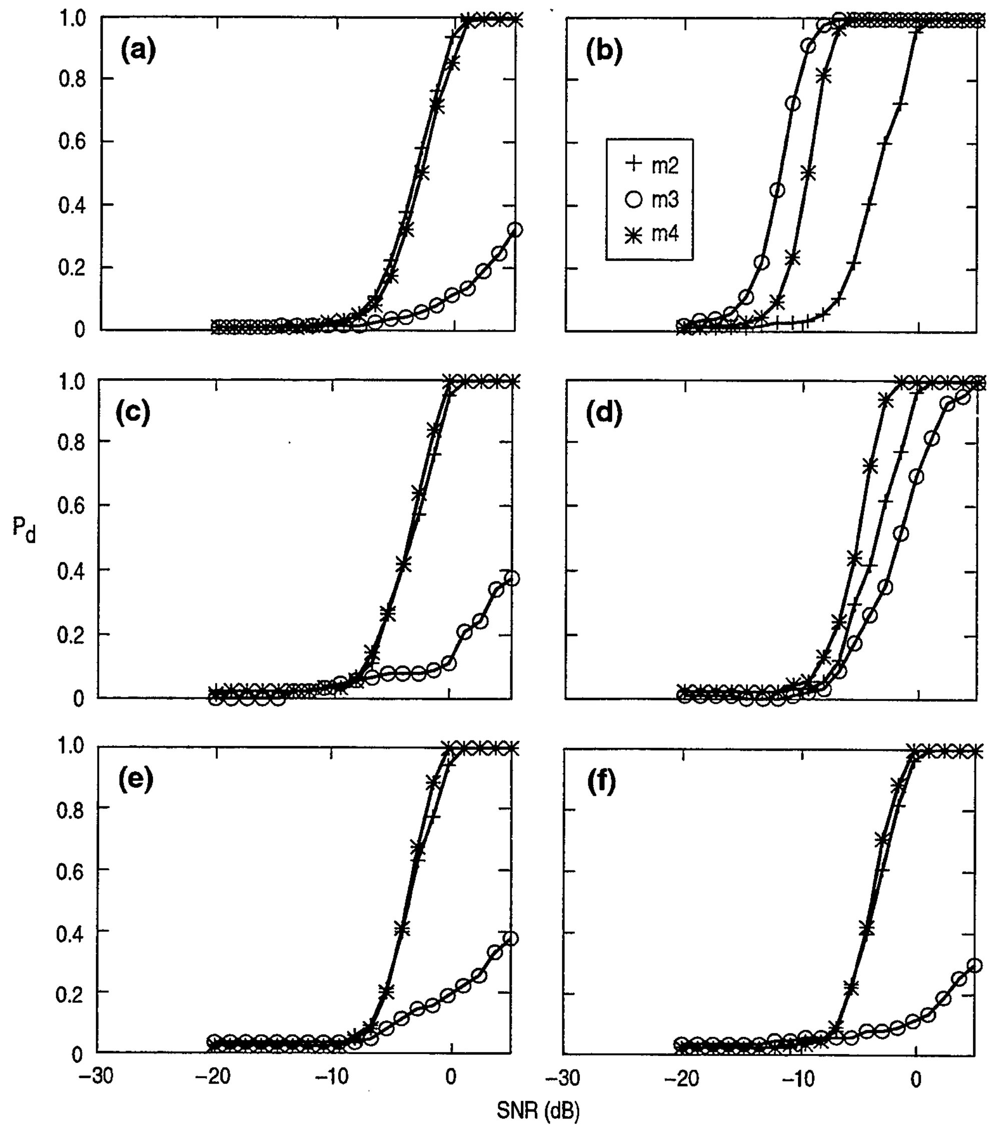

Fig. 12 — Single-channel detection simulations for the test signals in the SWellEX-3 noise, channel 43, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep

Simulation results using channel 43 noise (Fig. 12) are similar to the results using channel 2 noise in that the relative order of the moment detectors remains the same. This indicates that the nonGaussian behavior in the channel 2 noise has little effect on detection. However, detection is, in general, somewhat better in the channel 43 noise. One notable difference is that in the channel 43 noise, all three moment detectors achieve an SNR gain of about 1 dB for the narrow pulse.

To compare to the two channel detector simulations, ambient shipping noise from channels 2 and 43 are used in the moment calculations with $m^{x}_{4(2,2)}$ for the fourth order moment. The results are shown in Fig. 13. In this case, the second order moment detector clearly performs best for all of the test signals except the narrow pulse, which, like the single-channel cases with the measured noise, shows the third order moment detector performing best, followed by the fourth order moment detector. For the remaining five test signals, the second order moment detector performs better by

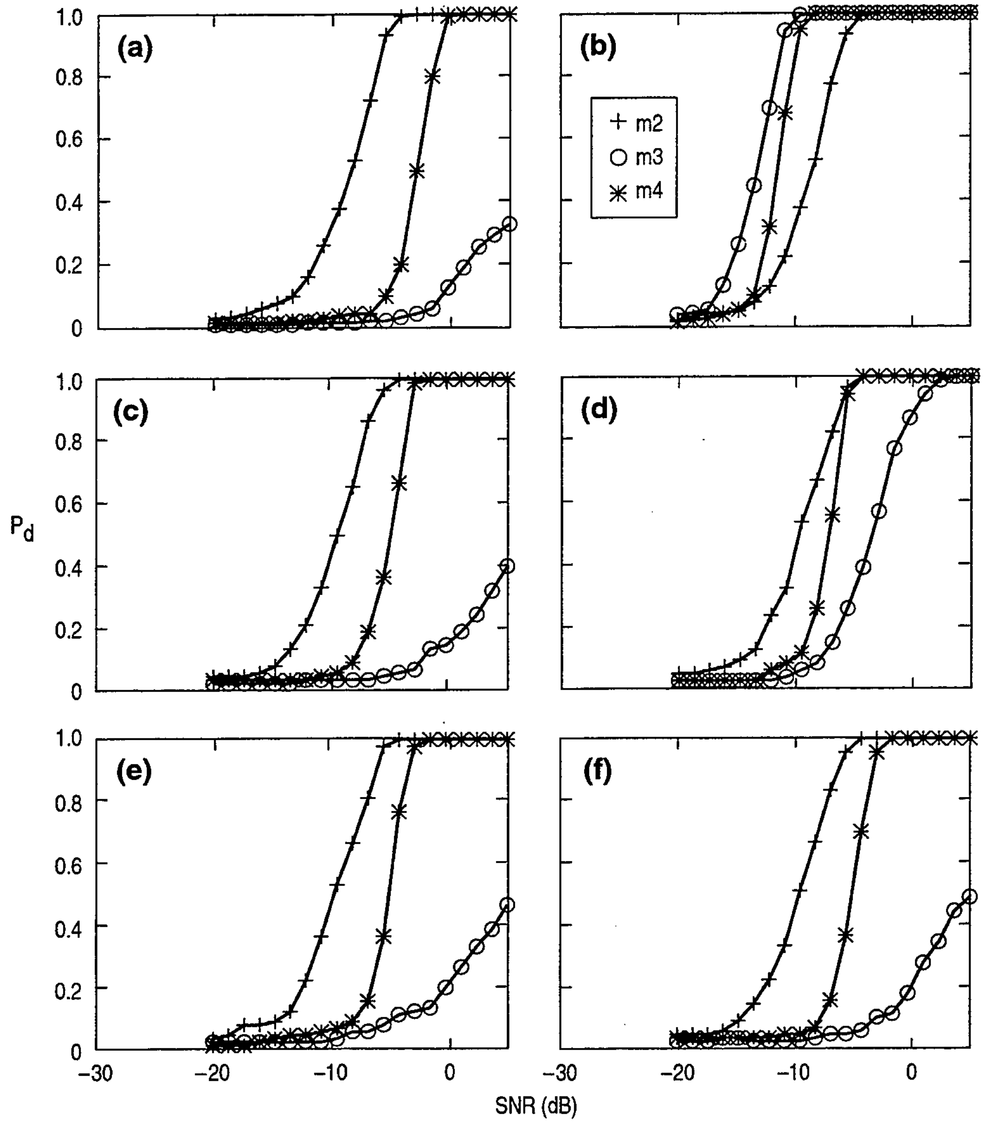

Fig. 13 — Two-channel detection simulations for the test signals in the SWellEX-3 noise. The fourth order moment detector is $m^{x}_{4(2,2)}$, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep.

about 2 to 6 dB over the fourth order moment detector, whereas it showed SNR gains of only 1.61 dB or smaller for the Gaussian noise. The second and fourth order moment detectors both improve when two channels of shipping data are used with $m^{x}_{4(2,2)}$ instead of one channel of data, although the second order moment detector improves considerably more than the fourth order, which is not the case with Gaussian noise.

If $m^{x}_{4(3,1)}$ is used for the fourth moment detector instead of $m^{x}_{4(2,2)}$, as shown in Fig. 14, then the fourth order moment detector improves considerably, although it only matches or surpasses the second order moment detector for 49- to 51-Hz sinusoid and the narrow pulse signals. The difference in detection results for $m^{x}_{4(2,2)}$ and $m^{x}_{4(3,1)}$ is an inherent result of the moment definitions. While the difference is slight for independent noise, it is amplified in the shipping noise, which is correlated across channels. This subject is discussed in App. B.

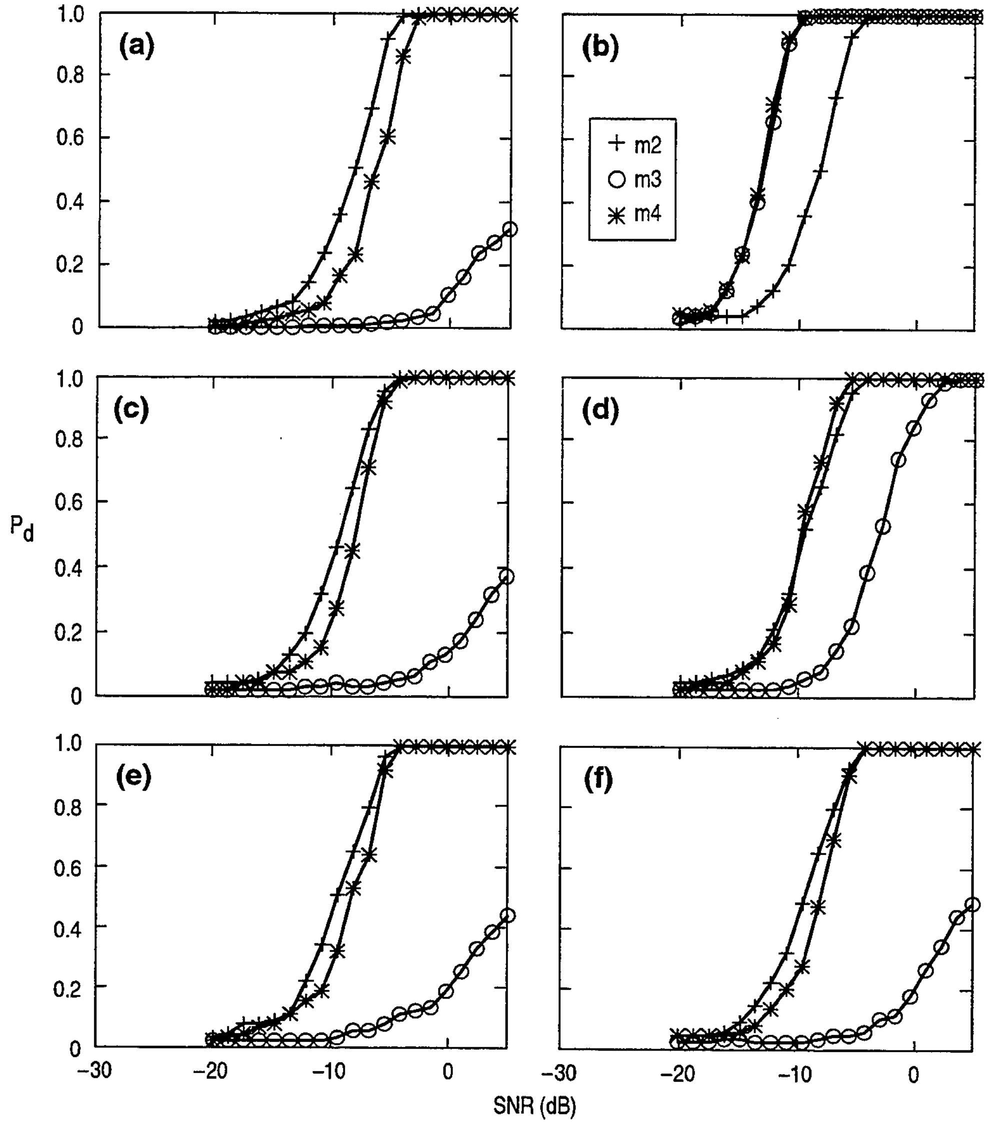

Fig. 14 — Two-channel detection simulations for the test signals in the SWellEX-3 noise. The fourth order moment detector is $m^x_{4_{(3,1)}}$, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep.

In the simulations depicted in Fig. 15, p channels of SWellEX-3 data are used for detection. In the second order detector, data from channels 2 and 43 are used, as shown in Figs. 12 and 13. In the third order detector, data from channel 16 are also used, and in the fourth order detector, data from channels 2, 43, 16, and 31 are used. The fourth order moment detector tends to improve with the additional channels of measured data, surpassing the performance of the second order moment detector over at least part of the P_d domain for all but the whale signal. For the narrow pulse, the SNR gain is approximately 8.73 dB and for the 49- to 51-Hz sinusoid, it is approximately 3.31 dB. This contrasts with the results for Gaussian noise, where the fourth order moment detector performs worse with p channels of data than it does with two channels.

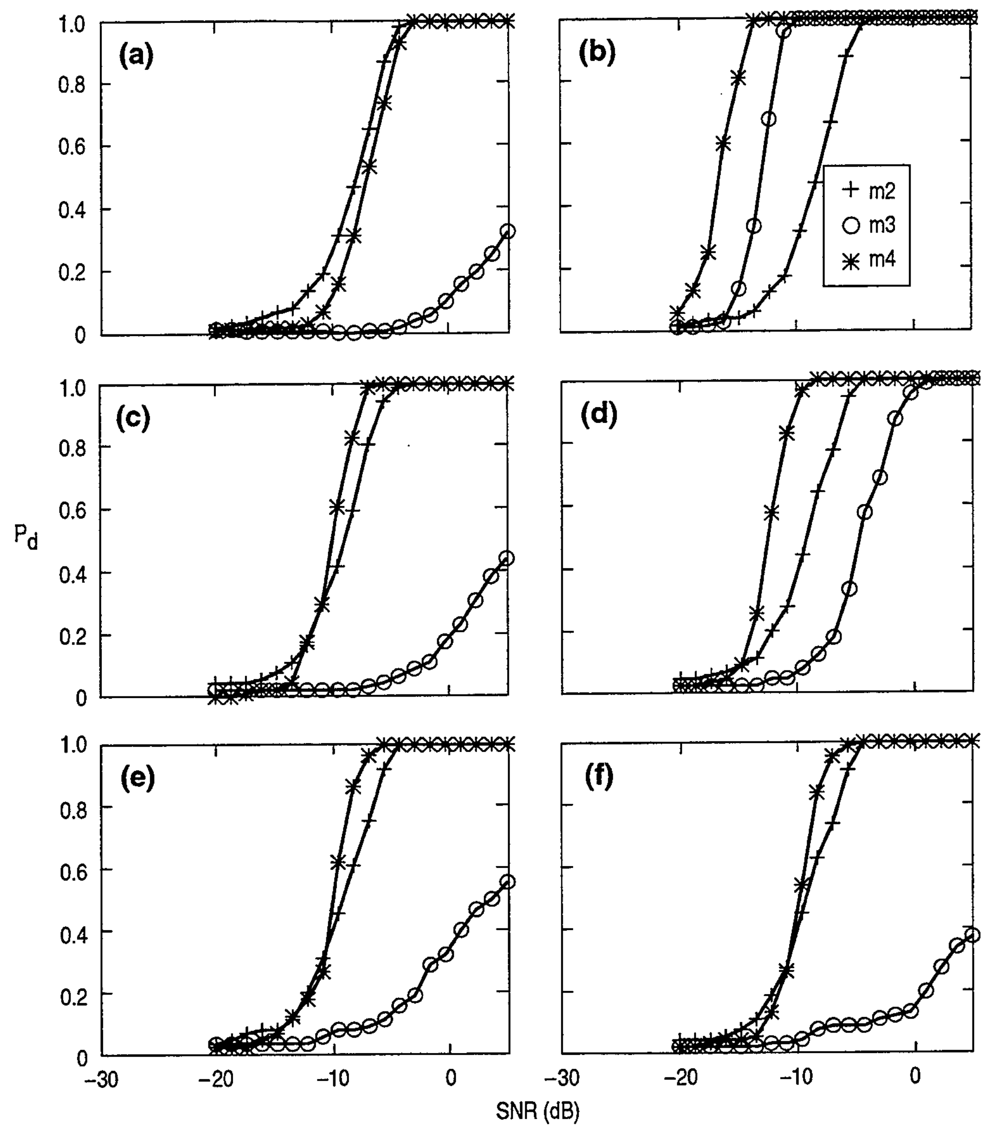

Fig. 15 — *p* channel detection simulations for the test signals in the SWellEX-3 noise, (a) whale, (b) narrow pulse, (c) 50-Hz sinusoid, (d) 49- to 51-Hz sinusoid, (e) linear FM sweep, and (f) nonlinear FM sweep

C. Effects of Prefiltering

Passbands for prefiltering are chosen based on visual inspection of the transient signal Fourier transforms shown in Fig. 6. As stated earlier, passband estimates may not be available for processing during passive detection. However, results from simulations that use a passband estimate are expected to be indicative of performance that might result from a more realistic application of prefiltering.

The single-channel detection results with prefiltering for Gaussian noise are shown in Fig. 16 for the six test signals. These results correspond to the results without prefiltering shown in Fig. 7. Prefiltering removes much of the noise passband for all the moment detectors, so overall performance is always improved with prefiltering, which does not disturb the signal. The fourth order moment detector improves with prefiltering and matches or slightly surpasses the performance

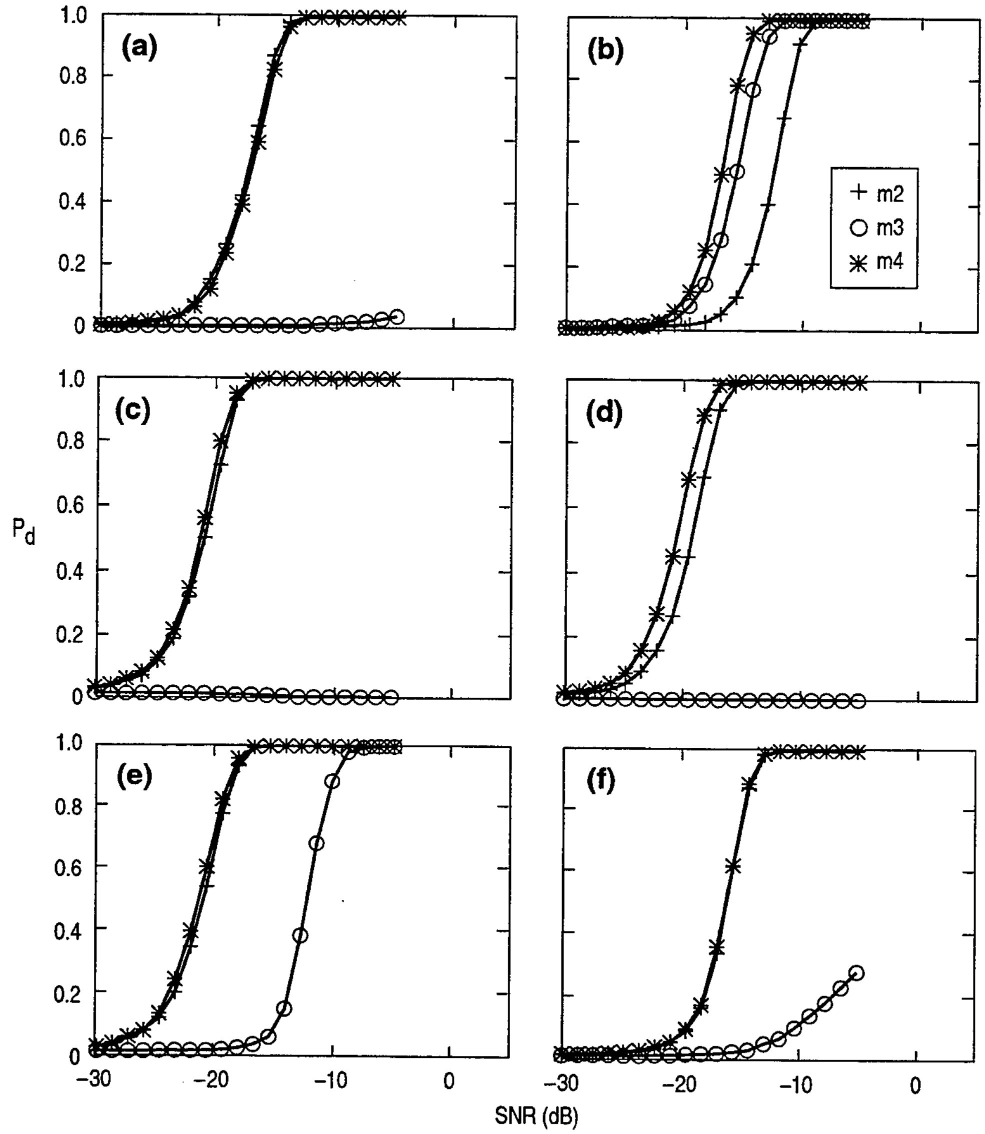

Fig. 16 — Single-channel detection simulations with prefiltering for the test signals in Gaussian noise, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz

of the second order moment detector for all the signals except the narrow pulse. The relative detection results for the narrow pulse signal show little change with prefiltering, probably due to the broadband nature of the signal.

When prefiltering is applied to the simulations with two channels of Gaussian noise and the fourth order moment detector defined by $m^{x}_{4(2,2)}$ (Fig. 17), the fourth order moment detector matches or performs best for the narrow pulse, 50-Hz sinusoid, 49- to 51-Hz sinusoid, and linear FM sweep signals. Without prefiltering (Fig. 8), the fourth order moment detector only performed better than the second order moment detector for the 49- to 51-Hz sinusoid and narrow pulse signals. When

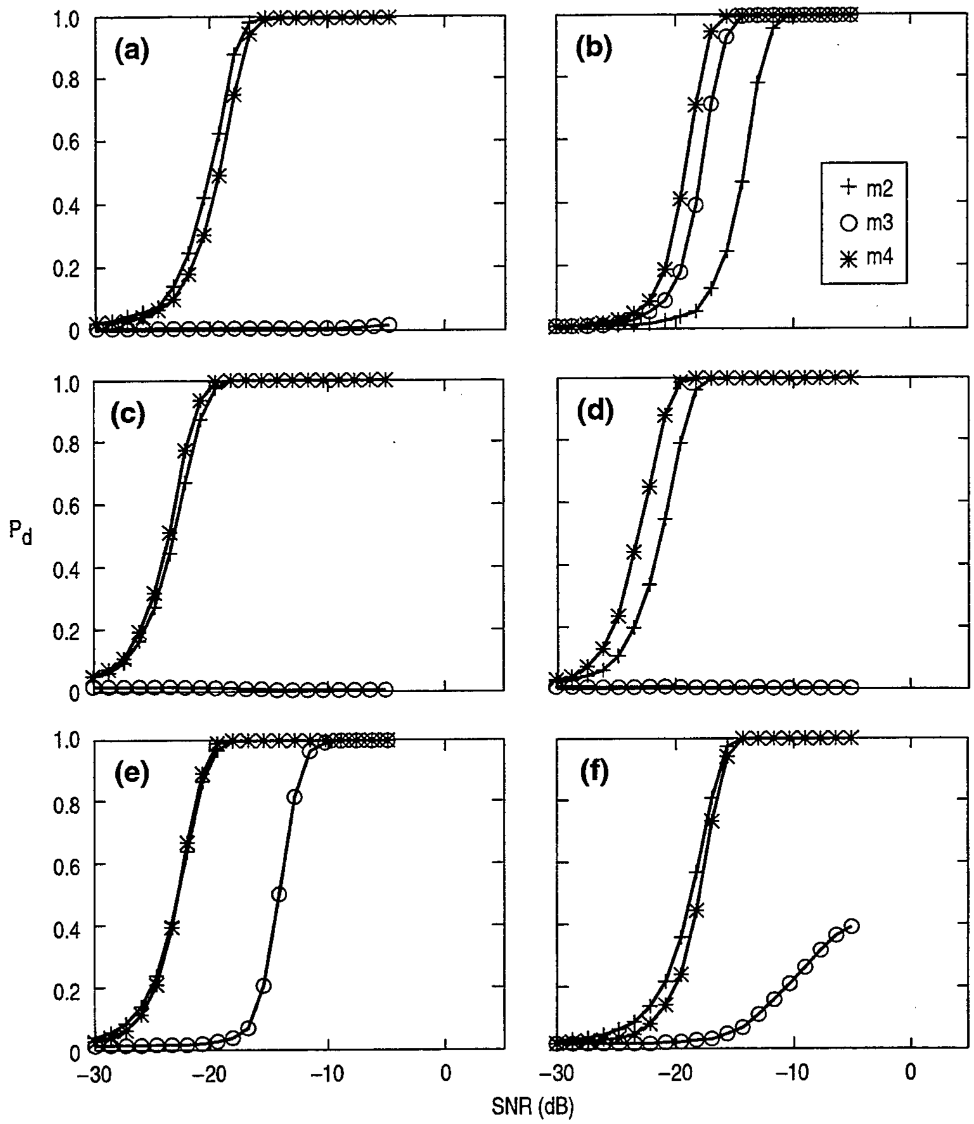

Fig. 17 — Two-channel detection simulations with prefiltering for the test signals in Gaussian noise. The fourth order moment detector is $m^x_{4(2,2)}$, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz.

$m^x_{4(3,1)}$ is instead used for the fourth order moment detector, as in Fig. 18, only minor changes appear for any of the test signals.

In Fig. 19, p channels of Gaussian noise are used in the simulations with prefiltering. Here, the fourth order moment detector surpasses the other detectors for all but the whale signal and the nonlinear FM sweep, in which case it matches the performance of the second order moment detector. Without prefiltering (Fig. 10), the fourth order moment surpassed the second order moment in detection performance for only the narrow pulse signal. This is a clear indication of the nonlinear improvement with moment order that prefiltering has on detector performance.

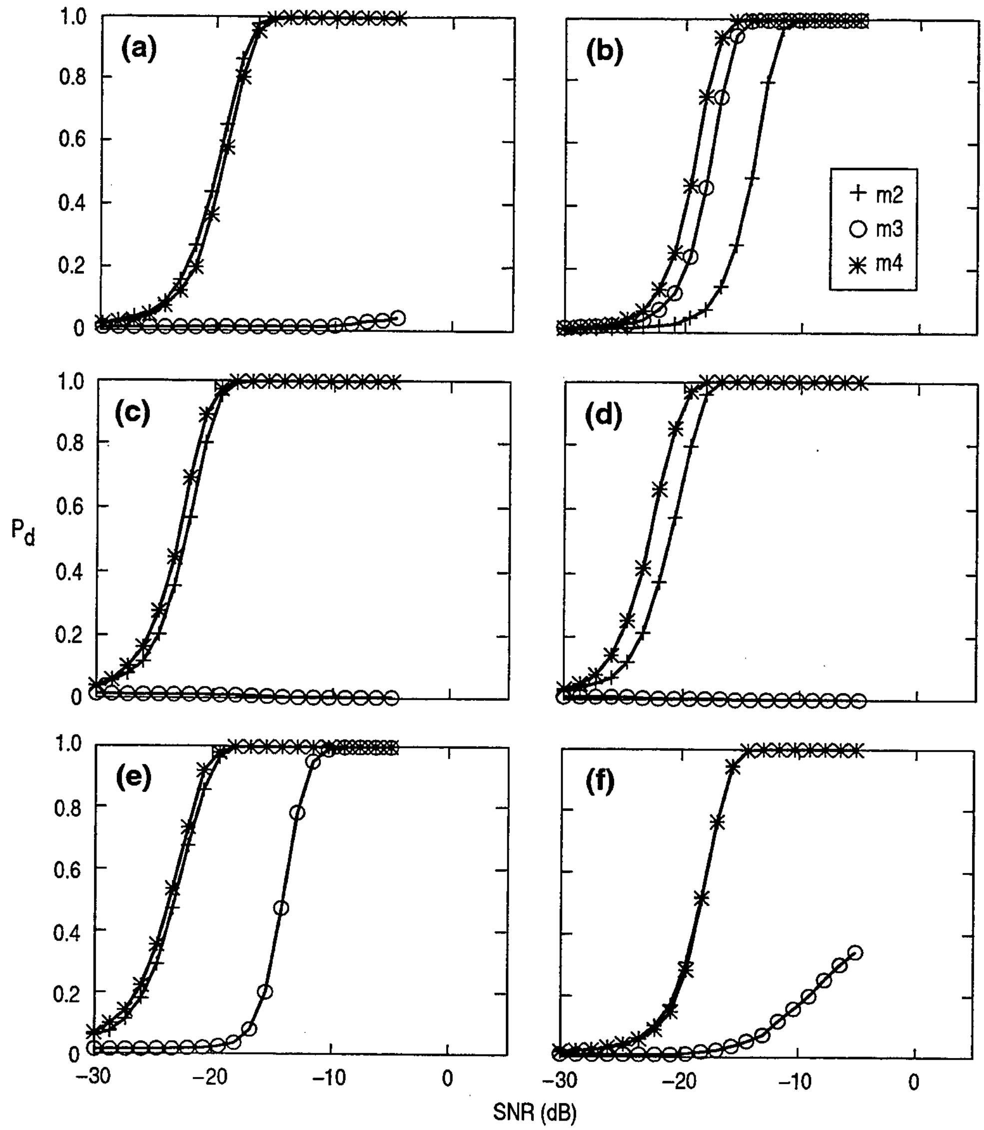

Fig. 18 — Two-channel detection simulations with prefiltering for the test signals in Gaussian noise. The fourth order moment detector is $m^x_{4_{(3,1)}}$, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz. and (f) nonlinear FM sweep 0–100 Hz.

Prefiltering with the SWellEX-3 channel 2 noise (single-channel case) has little effect on the relative ranking of detector performance except to show a slight increase for the fourth order moment relative to the second for some signals (see Fig. 20 and compare to Fig. 11). Similar results occur for the channel 43 noise (see Fig. 21 and compare to Fig. 12).

With prefiltering before detection when using two channels of the SWellEX-3 noise and $m^x_{4(2,2)}$ for the fourth order moment detector, the second order moment detector performed best for all but the narrow pulse signal and the 49- to 51-Hz sinusoid signal, the latter showing the fourth order moment detector slightly surpassing the second order moment detector for values of P_d above 0.5 (Fig. 22). The fourth order moment detector also greatly improves with prefiltering for the whale signal, but the second order moment detector still performs best by a small margin. As discussed in

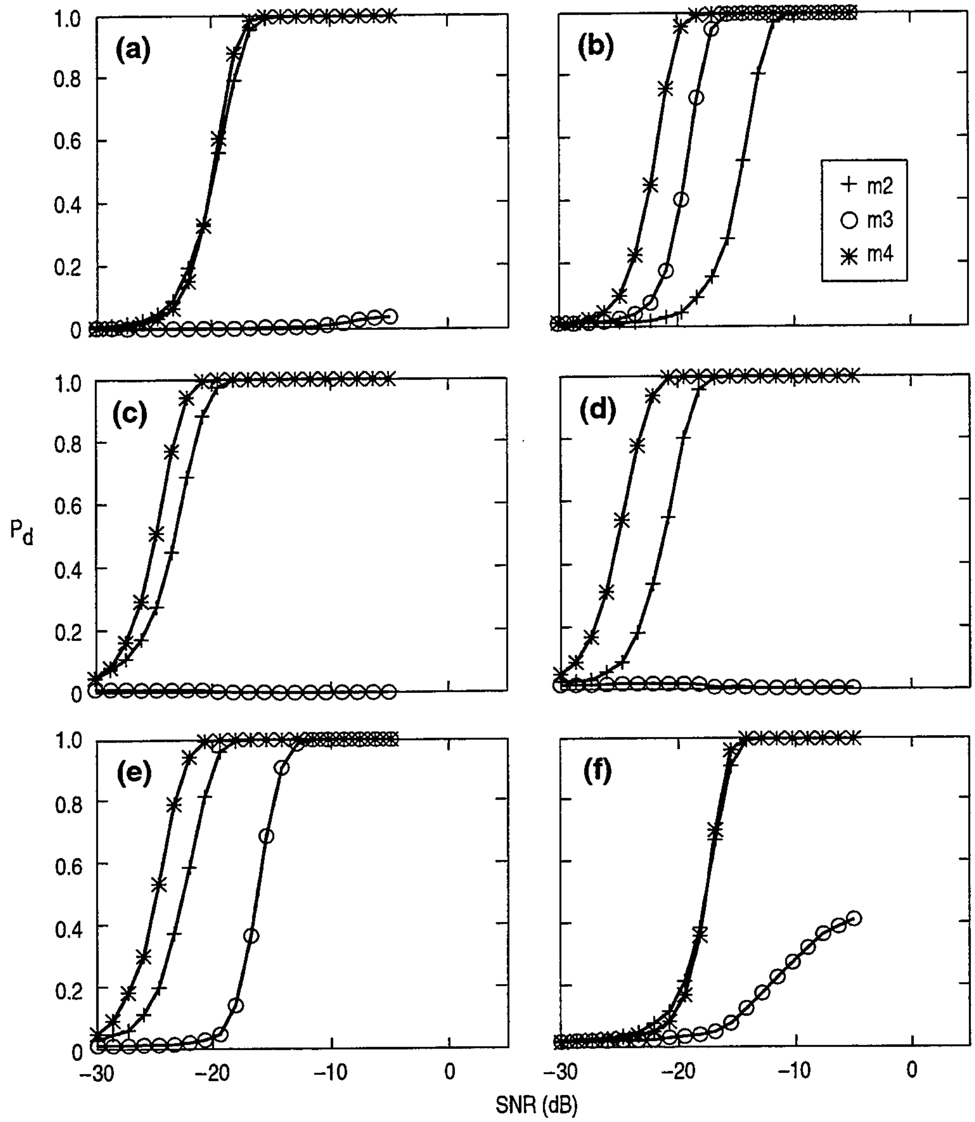

Fig. 19 — p channel detection simulations with prefiltering for the test signals in Gaussian noise, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz

App. B, the poor performance of the fourth order detector for $m^x_{4(2,2)}$ results from the interchannel correlation of the measured noise.

Figure 23 shows the results when $m^x_{4(3,1)}$ is used for the fourth order moment detector. Prefiltering causes the fourth order moment detector performance to match or exceed the others for all six test signals, whereas without prefiltering (Fig. 14), it matches or exceeds the other moments for only the narrow pulse and 49- to 51-Hz sinusoid signals. While little or no improvement occurs in the uncorrelated Gaussian noise when using $m^x_{4(3,1)}$ rather than $m^x_{4(2,2)}$, $m^x_{4(3,1)}$ is clearly the better choice in the correlated shipping noise.

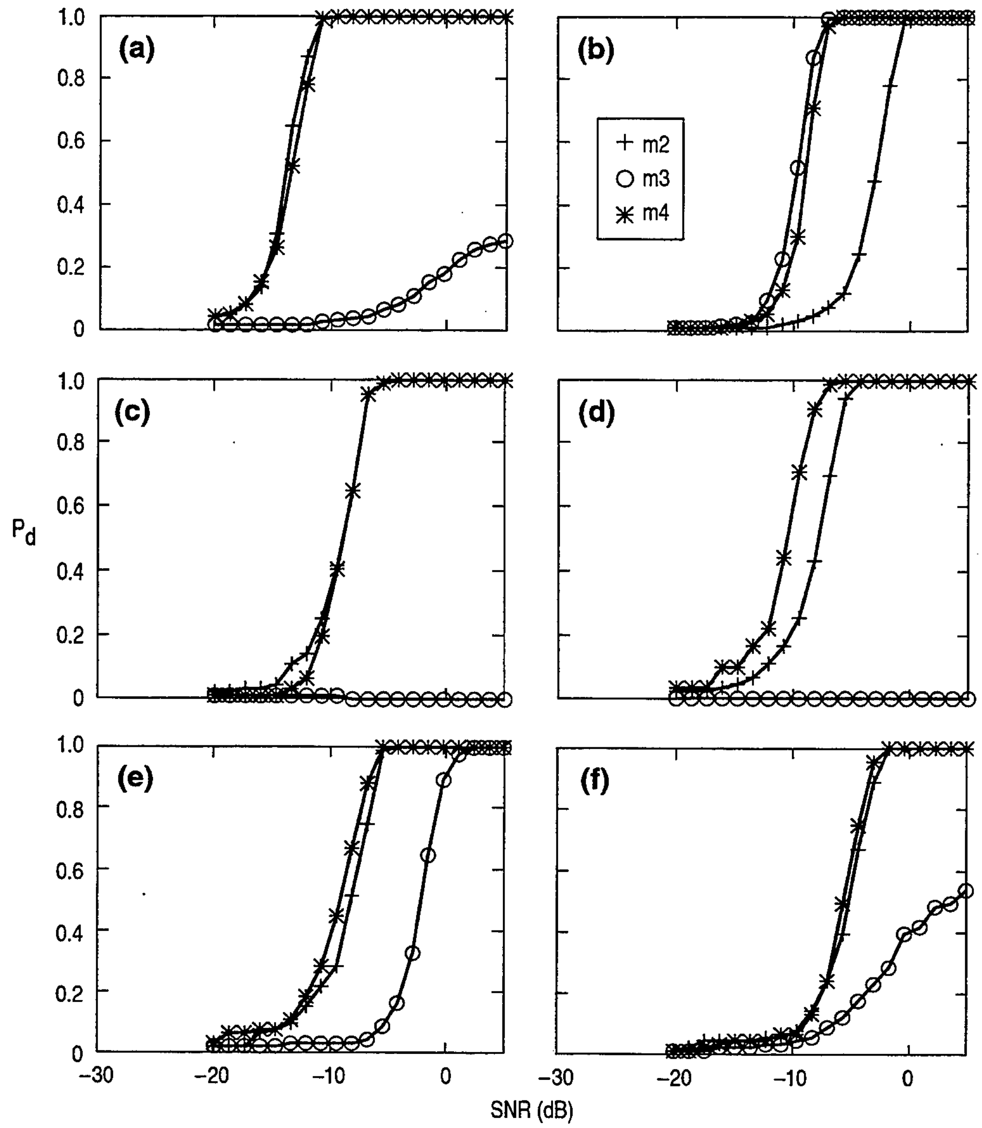

Fig. 20 — Single-channel detection simulations with prefiltering for the test signals in the SWellEX-3 noise, channel 2, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz

Finally, when two to four channels of SWellEX-3 noise are used in the moment detectors, as shown in Fig. 24, the fourth order moment detector significantly surpasses the second order moment detector for all but the whale signal, in which case the fourth order moment detector essentially matches the performance of the second order moment. These results indicate that the additional channel information included in the third and fourth order detectors is being used advantageously, except for the third order moment in cases where it is not suitable.

The third order moment detector shows virtually no detection ability with prefiltering and using one, two, or p channels of data for the whale, 50-Hz sinusoid, and 49- to 51-Hz sinusoid signals, which all have negative third order signal moments when prefiltered. This holds in simulations with the Gaussian noise, as well as with the measured shipping noise. Recall that the third order detector

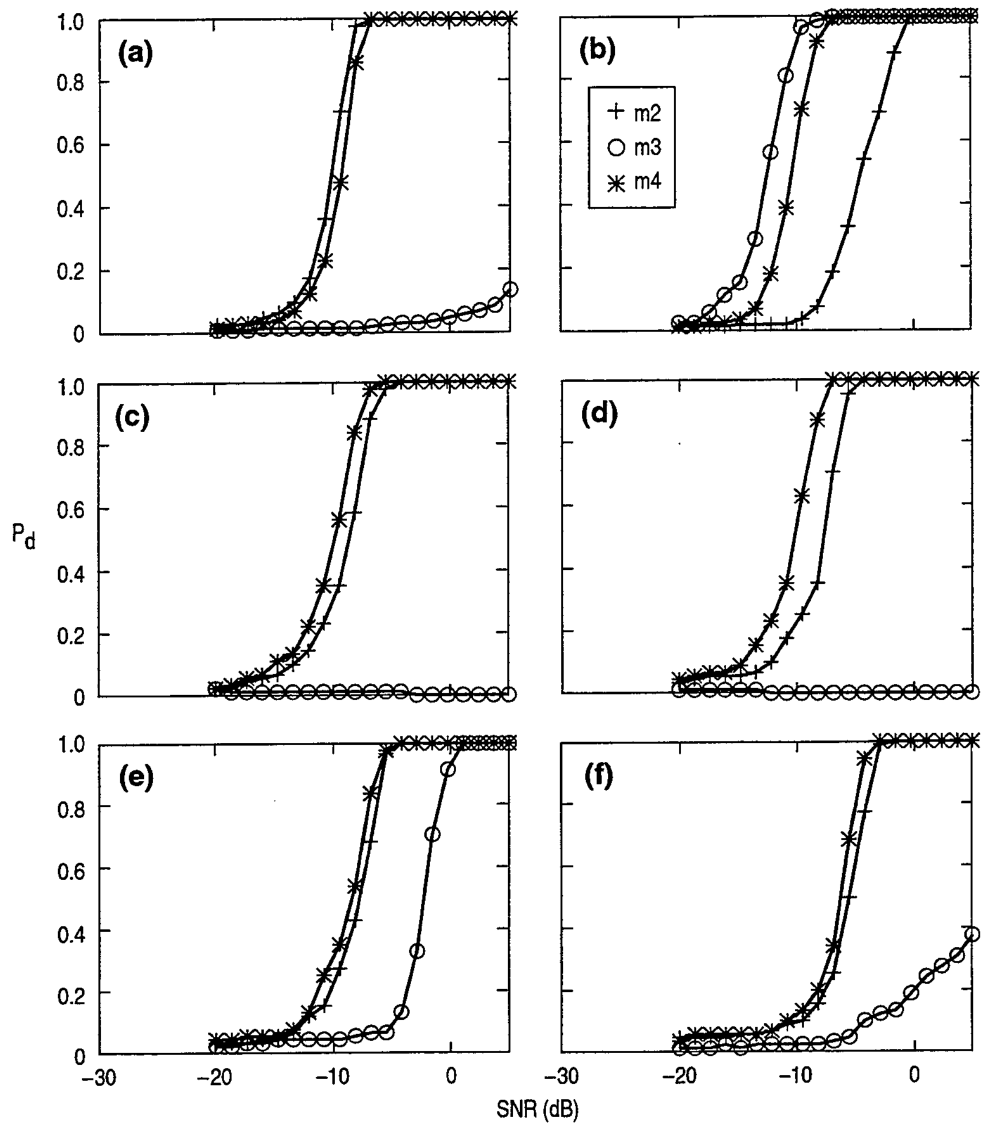

Fig. 21 — Single-channel detection simulations with prefiltering for the test signals in the SWellEX-3 noise, channel 43, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz

used in this study assumes a positive peak and, therefore, is failing to detect the negative third order moments of these signals. Although the third moment of the prefiltered nonlinear FM sweep is significantly larger than that of the linear FM sweep, the 40- to 60-Hz prefiltering passband used for the linear FM sweep filters out much more noise than the broader 0- to 100-Hz nonlinear FM sweep filter, causing the third order moment detector to perform better for the former.

D. Summary of Computer Simulations

Tables 3–5 contain summaries of the cases for which the third or fourth order moment detector performance matches (*M*) or surpasses (*S*) the performance of the second order detector over at

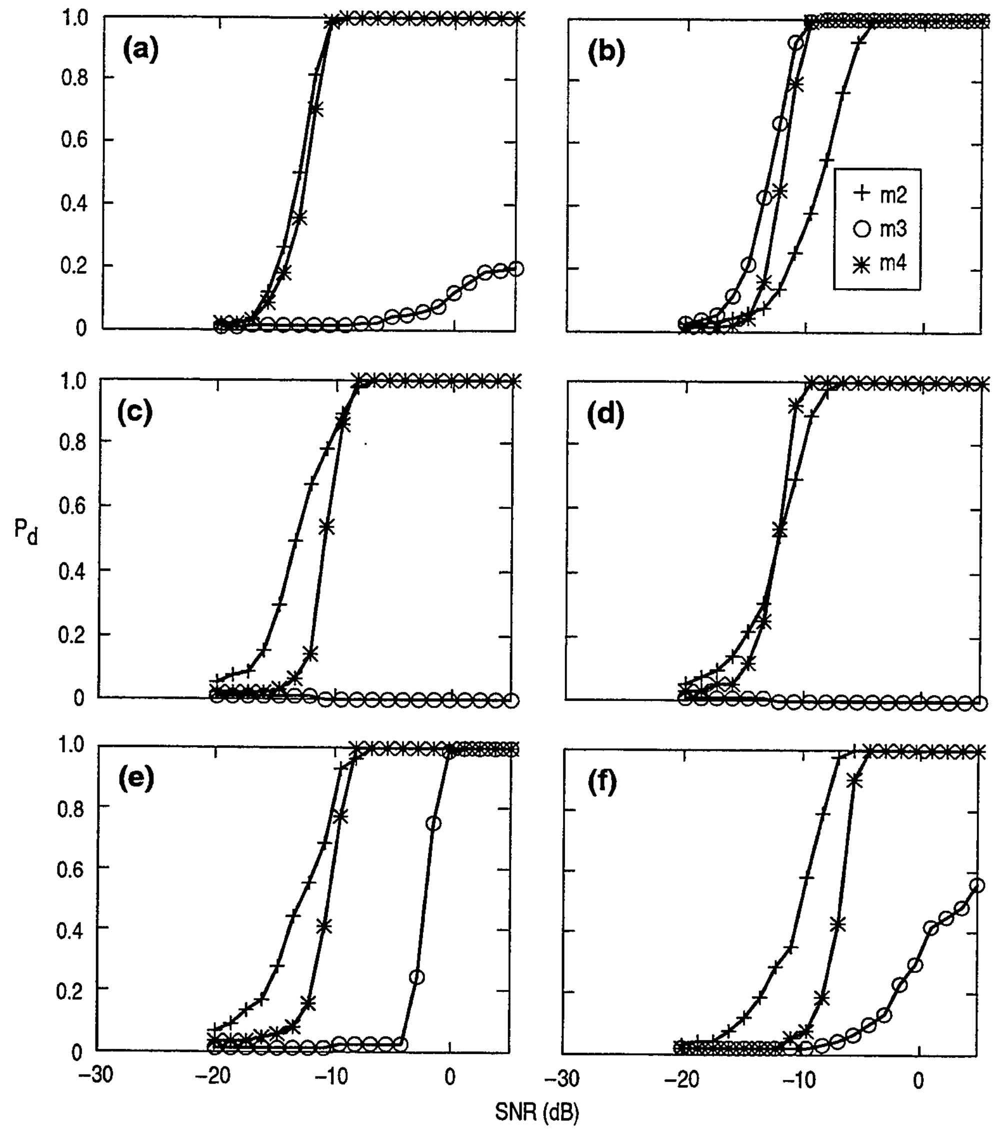

Fig. 22 — Two-channel detection simulations with prefiltering for the test signals in the SWellEX-3 noise. The fourth order moment detector is $m^x_{4(2,2)}$, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz.

least half of the P_d range for a single channel, two channels, and p channels of input data. The results are derived by inspection of the detection curves in Figs. 7–24. The detectors are said to match when the curves are within 0.5 dB. While the results for single-channel detection with either channel 2 or 43 are generally comparable, for the few instances when they differ slightly, the better higher order performance is utilized. Also, since the results when using $m^x_{4(3,1)}$ tend to be better than $m^x_{4(2,2)}$ for the measured noise, the former is represented in Table 4.

In Gaussian noise without prefiltering, the second order moment detector tends to perform best for the signals that are only slightly nonGaussian (whale, 50-Hz sinusoid, linear FM sweep, and nonlinear FM sweep). With prefiltering, the higher order detectors (in most cases, the tricorrelation detector) match or surpass the second order moment detector when p channels of noise are used.

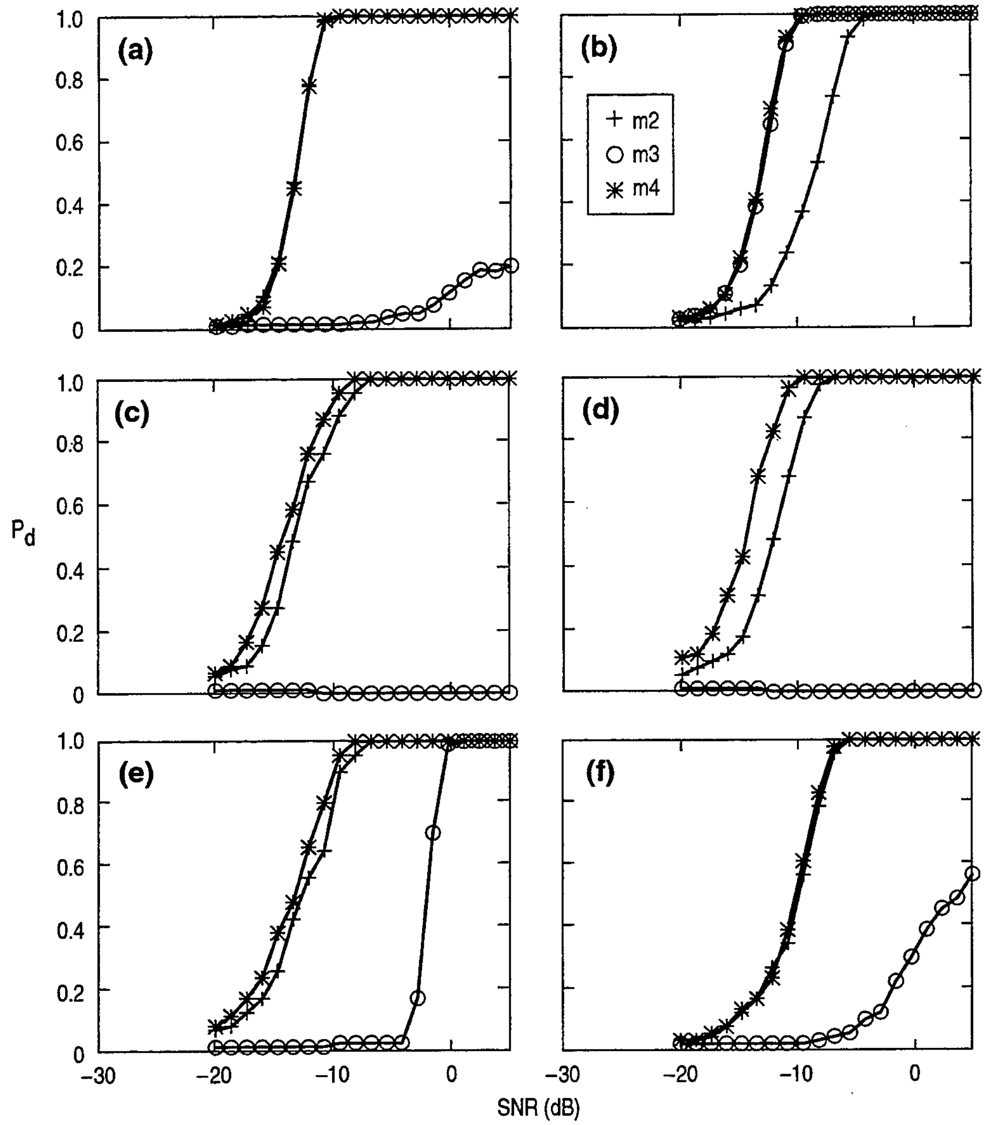

Fig. 23 — Two-channel detection simulations with prefiltering for the test signals in the SWellEX-3 noise. The fourth order moment detector is $m^x_{4_{(3,1)}}$, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz.

When one or p channels of SWellEX-3 data are used, the fourth and sometimes the third order moment detectors tend to match or perform better than the second order moment detector, both with and without prefiltering for the single-channel and p channel cases. When two channels of data are used, the second order moment detector tends to perform best for the signals that are only slightly nonGaussian, unless prefiltering is included.

Tables 3–5 give relative detector performance for individual cases, but not absolute detection levels. To assess the overall performance of the detectors in all cases and compare relative amounts of gain, numerical summary tables of the detection results are given in App. C in the same groupings given in Tables 3–5. Each of the detection curves shown in Figs. 7–24 is interpolated at $P_d = 0.5$, and the associated SNRs are listed in the tables, along with the third and fourth order moment

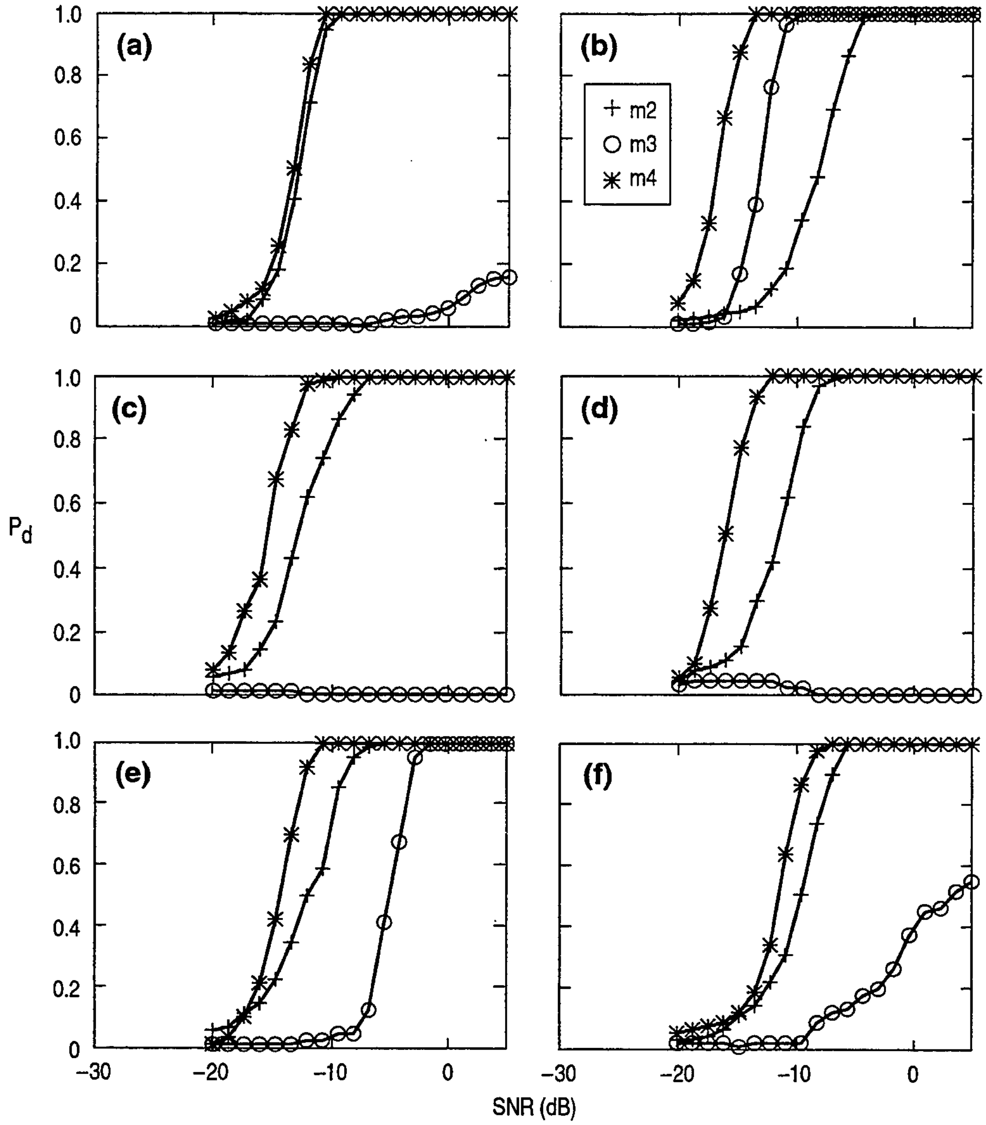

Fig. 24 — p channel detection simulations with prefiltering for the test signals in the SWellEX-3 noise, (a) whale 10–25 Hz, (b) narrow pulse 0–256 Hz, (c) 50-Hz sinusoid 45–55 Hz, (d) 49- to 51-Hz sinusoid 40–60 Hz, (e) linear FM sweep 40–60 Hz, and (f) nonlinear FM sweep 0–100 Hz

detector SNR gains, which are defined as the difference in decibels between the second order moment SNR at $P_d = 0.5$ and the third or fourth order moment SNR at $P_d = 0.5$. Thus, a positive SNR gain indicates that the third or fourth order moment detector performs better than the second order (detects at a lower SNR).

The highest SNR gains occur for the narrow pulse signal, which, of the six test signals, has the highest values of skewness and kurtosis. For the fourth order moment detector and one or multiple channels of input data, the SNR gains for Gaussian noise are between 3.47 and 5.97 dB without prefiltering and 4.40 and 7.54 dB with prefiltering for one, two, and p channels of noise, respectively. The third order moment gains tend to be somewhat lower. The 49- to 51-Hz sinusoid, which has the

Table 3 — Positive SNR Gains with a Single Channel of Data

SIGNAL	GAUSSIAN NOISE		SWellEX-3 NOISE	
	WITHOUT PREFILTERING	WITH PREFILTERING	WITHOUT PREFILTERING	WITH PREFILTERING
Whale	M	M	M	M
Narrow Pulse	S	S	S	S
50-Hz Sinusoid		M	M	S
49- to 51-Hz Sinusoid	M	S	S	S
Linear FM Sweep		M	M	S
Nonlinear FM Sweep		M	S	S

Table 4 — Positive SNR Gains with Two Channels of Data

SIGNAL	GAUSSIAN NOISE		SWellEX-3 NOISE	
	WITHOUT PREFILTERING	WITH PREFILTERING	WITHOUT PREFILTERING	WITH PREFILTERING
Whale		M		M
Narrow Pulse	S	S	S	S
50-Hz Sinusoid		S		S
49- to 51-Hz Sinusoid	M	S	M	S
Linear FM Sweep		M		S
Nonlinear FM Sweep		M		M

Table 5 — Positive SNR Gains with p Channels of Data

SIGNAL	GAUSSIAN NOISE		SWellEX-3 NOISE	
	WITHOUT PREFILTERING	WITH PREFILTERING	WITHOUT PREFILTERING	WITH PREFILTERING
Whale		M		M
Narrow Pulse	S	S	S	S
50-Hz Sinusoid		S	S	S
49- to 51-Hz Sinusoid	M	S	S	S
Linear FM Sweep		S	S	S
Nonlinear FM Sweep		M	S	S

next highest values of skewness and kurtosis, shows SNR gains between –0.04 and 0.34 dB without prefiltering and between 1.13 and 3.88 dB with prefiltering. In the measured noise, the fourth order moment gains for the narrow pulse are between 3.04 and 8.73 dB without prefiltering and between 4.59 and 8.68 dB with prefiltering. The corresponding values for the 49- to 51-Hz sinusoid are 0.21 and 3.32 dB without prefiltering and 2.36 and 4.52 dB with prefiltering. These results for the measured noise are for $m^x_{4(3,1)}$ rather than $m^x_{4(2,2)}$, which does not perform well.

If prefiltering is used, the overall best detection occurs when p channels of data are used in the moment calculations rather than one or two channels for both the Gaussian and measured noise. If prefiltering is not used, the overall best detection occurs when using p channels of data for the measured noise, but not for the Gaussian noise and the more Gaussian signals, which do best with two channels of data. In the cases where using p channels results in superior detection, the fourth order moment detector performs best for most signals.

VII. CONCLUSIONS

Comparisons of the performance of second through fourth order moment detectors for six test signals in simulated uncorrelated Gaussian noise and measured shallow-water ambient shipping noise indicate that the fourth order moment, in many cases, detects transient signals better than the second order moment, or energy detector. The third order moment detector performs better than the second for only one test signal, which is strongly nonGaussian. Differences in detector performances are found when the noise is measured shipping noise rather than simulated, uncorrelated Gaussian noise. The measured shipping noise is fairly Gaussian on three of the four channels, and correlated in both time and across channels. This suggests that the correlation of the measured noise is, perhaps, more influential than the Gaussianity.

In uncorrelated Gaussian noise, prefiltering is required for the higher order moment detectors to match or surpass the second order moment detectors for the test signals that are only slightly nonGaussian, whether one, two, or p channels of data are used. Prefiltering is not required for the two signals that are significantly nonGaussian. In contrast, in the shipping noise, the higher order moment detectors (usually the fourth order moment) match or surpass the second order moment for the two most nonGaussian signals and most of the more Gaussian signals when either one or p channels of data are used. When two channels of data are used, the second order moment tends to detect best for all but the most nonGaussian signals. With prefiltering, the higher order moment detectors perform best for most of the test signals, whether using one, two, or p channels of data.

Detection results indicate that using p channels of data in the moment calculation, rather than one or two channels, generally gives the best performance overall, and that the fourth order moment is usually the best detector in these cases. The exception is for Gaussian noise without prefiltering, in which case the use of two channels of data and the second order moment gives the best detection for the signals that are only slightly nonGaussian.

An interesting result of the analysis is that for one and two channels of input data, the third order moment detector performs as well as or better than the fourth order moment detector for the narrow pulse in the shipping noise, whereas the reverse is the case for the Gaussian noise. This indicates that the shipping noise nonGaussianity over the 0- to 256-Hz frequency band results more from the fourth order statistic than the third. The third order moment detector performs poorly for all but the most nonGaussian signals due to the potential for negative or small detection statistics.

Since its performance does not appear robust, its use is not recommended unless modifications are introduced.

A second interesting outcome of the analysis is that using $m^x_{4(3,1)}$ or $m^x_{4(2,2)}$ in the uncorrelated Gaussian noise results in similar detection performance. However, $m^x_{4(3,1)}$ clearly performs better than $m^x_{4(2,2)}$ in the correlated shipping noise. The reasons for this are discussed in App. B.

The application of prefiltering improves the performance of the third and fourth order moment detectors relative to the second order moment detector, both in Gaussian and measured shipping noise. These results indicate that applications of prefiltering that do not require knowledge of the signal passband should be investigated. These will be important when the signal passband is unknown and when many signals of different passbands are being detected.

The higher order moment detectors tend to perform well for the narrow pulse and 49- to 51-Hz sinusoid signal, which are significantly nonGaussian, but the gains for the other test signals, which are only slightly nonGaussian, are generally smaller. Although the higher order moment detectors are not expected to surpass the second order detector for near-Gaussian signals in Gaussian noise, the fourth order detector often performs as well as or better than the second order detector for these signals when prefiltering is used. In the correlated noise, it performs as well as or better both with and without prefiltering.

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Appendix A

DEPENDENCE OF DETECTION PERFORMANCE ON NOISE MOMENTS FOR SINGLE AND MULTIPLE CHANNELS OF DATA

The performance of moment detectors can be determined by a receiver operating characteristic (ROC) curve, which is a plot of P_d vs. P_{fa} for a given SNR, and is defined by the signal-absent (*s-a*) and signal-present (*s-p*) PDFs for the detector (see Fig. A1). The PDFs are normalized to have unit area. For a given threshold, the area to the right beneath the *s-p* PDF is the P_d and the area to the right beneath the *s-a* PDF is the P_{fa}. For example, if the threshold is fixed at m_p^s, denoted by the dashed line in the figure, the P_d is 0.5 and the area to the right of m_p^s is the corresponding P_{fa}. Decreasing the SNR broadens the PDFs and corresponds to larger P_{fa}, or poorer detection. Thus, detection performance is determined by the choice of threshold and the respective centers and shapes of the *s-a* and *s-p* PDFs, each of which is defined by its moments of all orders. For zero-mean Gaussian PDFs, the shapes are completely determined by the PDF variance (or second moment).

When p channels of data are used in the *p-th* order moment detector, the *s-p* and *s-a* *q-th* order PDF moments are

$$M_p^q(s-p) = E\left\{\left[\sum_{k=0}^{N-1}\left[s(t)+n_1(t)\right]\left[s(t)+n_2(t)\right]\cdots\left[s(t)+n_p(t)\right]\Delta t\right]^q\right\} \tag{A1}$$

$$M_p^q(s-a) = E\left\{\left[\sum_{k=0}^{N-1} n_1(t)n_2(t)\cdots n_p(t)\Delta t\right]^q\right\} . \tag{A2}$$

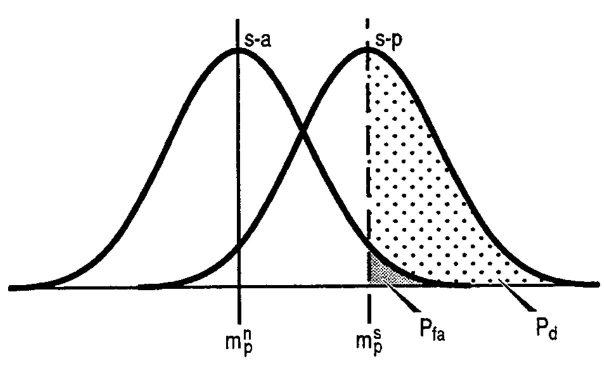

Fig. A1 — Parameters that define a ROC curve

When fewer than p channels of data are used, some of the noise sequences, $n_i(t)$, in Eqs. (A1) and (A2) are the same. For simplicity, assume that the noise is zero mean. The expression for $M_p^q(s-p)$ upon expansion gives

$$M_p^q(s-p) = (\Delta t)^q E\left\{\begin{aligned}&\left[\sum_{k_1=0}^{N-1}\left[s(t_1)+n_1(t_1)\right]\left[s(t_1)+n_2(t_1)\right]\cdots\left[s(t_1)+n_p(t_1)\right]\right]\\ &\bullet\left[\sum_{k_2=0}^{N-1}\left[s(t_2)+n_1(t_2)\right]\left[s(t_2)+n_2(t_2)\right]\cdots\left[s(t_2)+n_p(t_2)\right]\right]\\ &\bullet\cdots\left[\sum_{k_q=0}^{N-1}\left[s(t_q)+n_1(t_q)\right]\left[s(t_1)+n_2(t_q)\right]\cdots\left[s(t_1)+n_p(t_q)\right]\right]\end{aligned}\right\}, \tag{A3}$$

or

$$M_p^q(s-p) = (\Delta t)^q E\left\{\begin{aligned}&\left[\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\cdots\sum_{k_q=0}^{N-1} s(t_1)\,s(t_2)\cdots s(t_q)\right]\\ &+\left[\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\cdots\sum_{k_q=0}^{N-1}\left\{\begin{aligned}&\left(n_1(t_1)\,n_2(t_1)\cdots n_p(t_1)\right)\\ &\left(n_1(t_2)\,n_2(t_2)\cdots n_p(t_2)\right)\\ &\bullet\cdots\left(n_1(t_q)\,n_2(t_q)\cdots n_p(t_q)\right)\end{aligned}\right\}\right]\\ &+\left[\begin{aligned}&\text{cross terms of the form}\\ &\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\cdots\sum_{k_q=0}^{N-1} s^R(t)\,n_1^{r_1}\left(t_{i_1}\right)n_2^{r_2}\left(t_{i_2}\right)\cdots n_p^{r_p}\left(t_{i_p}\right)\end{aligned}\right]\end{aligned}\right\} \tag{A4}$$

where $R \neq 0, p$; $p = R + r_1 + r_2 \ldots + r_p$; and $i_j \in [1, 2, \ldots, q]$. Consider the three terms in square brackets in Eq. (A4). If p channels of data are used for moment detection, then the first term is the general *q-th* order signal moment summed over all combinations of times. The second term reduces to $M_p^q(s-a)$ (Eq. (A2)) even if fewer than p channels of data are used. The same dependence that the *s-a* PDF moments have on the noise moments also occurs in the *s-p* PDF moments. The remaining cross terms are small, but nonzero for finite sequences of data.

When p channels of data are used in the p-th order moment detector, expansion of the expression for $M_p^q(s-a)$ in Eq. (A2) gives

$$M_p^q(s-a) = (\Delta t)^q E \left\{ \begin{array}{l} \left[\sum_{k_1=0}^{N-1} n_1(t_1)\, n_2(t_1) \cdots n_p(t_1) \right] \left[\sum_{k_2=0}^{N-1} n_1(t_2)\, n_2(t_2) \cdots n_p(t_2) \right] \\ \bullet \cdots \left[\sum_{k_q=0}^{N-1} n_1(t_q)\, n_2(t_q) \cdots n_p(t_q) \right] \end{array} \right\} . \tag{A5}$$

For large N and i.i.d. noise, this expression is approximately zero when q is odd. It is only nonzero when q is even and times are equal in pairs. Using delta function notation, the moments are nonzero when

$$(\Delta t)^q E \left\{ \begin{array}{l} \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \cdots \sum_{k_q=0}^{N-1} n_1(t_1)\, n_1(t_2) \cdots n_1(t_q)\, n_2(t_1)\, n_2(t_2) \cdots n_2(t_q) \\ \bullet\, n_p(t_1)\, n_p(t_2) \cdots n_p(t_q) \cdots \left[\sum \delta\left(t_{j_1} - t_{j_2}\right) \cdots \delta\left(t_{j_{q-1}} - t_{j_q}\right) \right] \end{array} \right\} \tag{A6}$$

is nonzero, where the summation over the product of delta functions is taken over all possible ways of dividing q integers into $q/2$ combinations of pairs. There are $(1)(3)(5)...(q-3)(q-1)$ terms in the summation. Evaluating the delta summation and using the independence of the noise results in

$$\begin{aligned} &\left[(1)\,(3)\,(5) \cdots (q-3)\,(q-1)\right] (\Delta t)^q N^{q/2} E\left\{ n_1^2(t_1)\, n_1^2(t_2) \cdots n_1^2(t_{q/2}) \right\} \\ &\bullet E\left\{ n_2^2(t_1)\, n_2^2(t_2) \cdots n_2^2(t_{q/2}) \right\} \cdots E\left\{ n_p^2(t_1)\, n_p^2(t_2) \cdots n_p^2(t_{q/2}) \right\} . \end{aligned} \tag{A7}$$

Since the square of the noise is uncorrelated when the noise is independent, this is equivalent to

$$\begin{aligned} &\left[(1)\,(3)\,(5) \cdots (q-3)\,(q-1)\right] (\Delta t)^q N^{q/2} E\left\{ n_1^2(t_1) \right\} E\left\{ n_1^2(t_2) \right\} \cdots E\left\{ n_1^2(t_{q/2}) \right\} \\ &\bullet E\left\{ n_2^2(t_1) \right\} E\left\{ n_2^2(t_2) \right\} \cdots E\left\{ n_2^2(t_{q/2}) \right\} \\ &\bullet \cdots E\left\{ n_p^2(t_1) \right\} E\left\{ n_p^2(t_2) \right\} \cdots E\left\{ n_p^2(t_{q/2}) \right\} \\ &= \left[(1)\,(3)\,(5) \cdots (q-3)\,(q-1)\right] (\Delta t)^q N^{q/2} \left[E\left\{ n^2(t) \right\} \right]^{pq/2} , \end{aligned} \tag{A8}$$

which is the even *q-th* order ensemble moment of the correlation of p zero-mean noise sequences. These powers of the moments are consistent with the moment relationships for a Gaussian distribution and depend only on the second order moment of the noise. This result has already been documented by Pflug et al. (1995b), but is included here for completeness.

If two channels of data are used in the *p-th* order moments and repeated such that for $p > 2$ ($p = 2$ is a p channel case), $p = p_1 + p_2$, with p_1 and p_2 equal to the number of times that each channel of data is repeated in the moment, then $M_p^q(s-a)$ can be written as

$$M_p^q(s-a) = E\left\{ \begin{array}{l} \left[\sum_{k_1=0}^{N-1} n_1^{p_1}(t_1) n_2^{p_2}(t_1)\Delta t\right]\left[\sum_{k_2=0}^{N-1} n_1^{p_1}(t_2) n_2^{p_2}(t_2)\Delta t\right] \\ \cdots\left[\sum_{k_q=0}^{N-1} n_1^{p_1}(t_q) n_2^{p_2}(t_q)\Delta t\right] \end{array} \right\}, \tag{A9a}$$

$$= (\Delta t)^q E\left\{ \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} \cdots \sum_{k_q=0}^{N-1} n_1^{p_1}(t_1)\, n_1^{p_1}(t_2) \cdots n_1^{p_1}(t_q)\, n_2^{p_2}(t_1)\, n_2^{p_2}(t_2) \cdots n_2^{p_2}(t_q) \right\}. \tag{A9b}$$

If either p_1 or p_2, or both p_1 and p_2 are odd, then expectation expression(s) involving the odd p_1 or p_2 in Eq. (A9b) will be nonzero only when q is even and times are equal in pairs, as given by the summation of delta functions in Eq. (A6) for the p channel case. After application of the appropriate delta functions, Eq. (A9b) reduces to

$$\left[(1)(3)(5)\cdots(q-3)(q-1)\right](\Delta t)^q N^{q/2} E\left\{ n_1^{p_1}(t_1)\, n_1^{p_1}(t_2) \cdots n_1^{p_1}(t_{q/2}) \right\}, \tag{A10a}$$
$$\bullet\, E\left\{ n_2^{p_2}(t_1)\, n_2^{p_2}(t_2) \cdots n_2^{p_2}(t_{q/2}) \right\}$$

and finally to

$$\left[(1)(3)(5)\cdots(q-3)(q-1)\right](\Delta t)^q N^{q/2} \left[E\left\{n^{p_1}(t)\right\}\right]^{q/2} \left[E\left\{n^{p_2}(t)\right\}\right]^{q/2}. \tag{A10b}$$

If p_1 and p_2 are both even, then Eq. (A9b) is nonzero for all times and q even or odd, and $M_p^q(s-a)$ is

$$(\Delta t)^q N^q \left[E\left\{n^{p_1}(t)\right\}\right]^q \left[E\left\{n^{p_2}(t)\right\}\right]^q. \tag{A11}$$

In the case where only a single channel of data is used in a *p-th* order moment detector, $M_p^q(s-a)$ reduces to

$$M_p^q(s-a) = (\Delta t)^q E\left\{\left[\sum_{k_1=0}^{N-1} n^p(t_1)\right]\left[\sum_{k_2=0}^{N-1} n^p(t_2)\right]\cdots\left[\sum_{k_q=0}^{N-1} n^p(t_q)\right]\right\}, \tag{A12a}$$

or

$$(\Delta t)^q E\left\{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\cdots\sum_{k_q=0}^{N-1} n^p(t_1)\, n^p(t_2)\cdots n^p(t_q)\right\}. \tag{A12b}$$

If p is odd, then Eq. (A12b) will be nonzero only when q is even and times are equal in pairs. Then $M_p^q(s-a)$ becomes

$$\left[(1)\,(3)\,(5)\cdots(q-3)\,(q-1)\right](\Delta t)^q N^{q/2}\left[E\left\{n^p(t)\right\}\right]^{q/2}. \tag{A13}$$

If, however, p is even, then Eq. (A12b) is nonzero for all times and all values of q, and $M_p^q(s-a)$ is

$$(\Delta t)^q N^q\left[E\left\{n^p(t)\right\}\right]^q. \tag{A14}$$

As mentioned above, the *s-p* PDF in any case is influenced by the number of data channels through the cross terms of Eq. (A4) when the signal sequences are finite.

Appendix B

DIFFERENCES BETWEEN THE FOURTH ORDER MOMENT DETECTORS FOR INDEPENDENT AND CORRELATED NOISE

To compare the different detection results for $m^{x}_{4(2,2)}$ in the independent and spatially correlated noise, one must consider the first and second moments, or mean and variance, of the *s-a* and *s-p* PDFs. While the higher order moments of the PDFs may contribute to detection performance, their effects are small since the PDFs are near Gaussian.

First consider the *s-p* PDF moments for $m^{x}_{4(2,2)}$, given by the ensemble average

$$M^{q}_{4}(s-p) = E\left\{\left[\Delta t \sum_{k=0}^{N-1}\left(\begin{array}{l} s^{4}(t) + n_1^2(t)n_2^2(t) \\ + 2s^{3}(t)\,n_1(t) + 2s^{3}(t)\,n_2(t) + s^{2}(t)\,n_1^2(t) + s^{2}(t)\,n_2^2(t) \\ + 2s(t)\,n_1^2(t)\,n_2(t) + 2s(t)\,n_1(t)\,n_2^2(t) + 4s^{2}(t)\,n_1(t)\,n_2(t) \end{array}\right)\right]^{q}\right\} \tag{B1}$$

with $x_1(t) = s(t) + n_1(t)$ and $x_2(t) = s(t) + n_2(t)$. $q = 1$ defines $m^{x}_{4(2,2)}$ (Eq. (2c)), which is the *s-p* PDF mean. Several of the terms in this expression contain a single noise factor (terms 3, 4, 5, and 6) that cannot reflect any spatial correlation between phones and contribute to the differences in detection. Three additional terms (terms 7, 8, and 9) contain one noise factor raised to an odd power, which can cause the terms to be either positive or negative for zero mean noise and, therefore, are small on average. The remaining term containing noise in the expectation of Eq. (B1) is the term $\Delta t \sum_{k=0}^{N-1} n_1^2(t)n_2^2(t)$, which is also the only term contributing to the *s-a* PDF moments. This term is essentially responsible for the differences found between detection in independent noise and detection in correlated noise.

The effect of this term on the *s-a* and *s-p* PDF means is equal for a given type of noise, and does not result in the detection differences found in the independent and correlated noise. However, consider the effect of this term on the variances of the *s-a* and *s-p* PDFs. The general expression for the *s-a* PDF moments for $m^x_{4(2,2)}$ is given by

$$M_4^q(s-a) = E\left\{\left[\sum_{k_1=0}^{N-1} n_1^2(t_1)\,n_2^2(t_1)\Delta t\right]\left[\sum_{k_2=0}^{N-1} n_1^2(t_2)\,n_2^2(t_2)\Delta t\right]\cdots\left[\sum_{k_q=0}^{N-1} n_1^2(t_q)\,n_2^2(t_q)\Delta t\right]\right\}. \quad \text{(B2)}$$

This is Eq. (A9a) from App. A with $p = 4$ and $p_1 = p_2 = 2$. It can be rewritten in the more convenient form of Eq. (A9b)

$$M_4^q(s-a) = (\Delta t)^q E\left\{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\cdots\sum_{k_q=0}^{N-1} n_1^2(t_1)\,n_1^2(t_2)\cdots n_1^2(t_q)\,n_2^2(t_1)n_2^2(t_2)\cdots n_2^2(t_q)\right\}. \quad \text{(B3)}$$

The second moment of the *s-a* PDF, which is related to the variance of the PDF, is found by setting $q = 2$ in Eq. (B3), i.e.,

$$M_4^2(s-a) = (\Delta t)^2 E\left\{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} n_1^2(t_1)\,n_1^2(t_2)\,n_2^2(t_1)\,n_2^2(t_2)\right\}. \quad \text{(B4)}$$

To compare detection in independent and correlated noise, assume that the average SNRs of the two types of noise and, therefore, the average variances of the noise, are equal. If $n_1^2(t)$ and $n_2^2(t)$ are correlated in the positive sense, then the summations in Eq. (B4) will be larger for the correlated noise than for the independent noise. Thus, the effects of the correlated noise are much more significant, i.e., the contributions are larger in the second moment of the *s-a* PDF than they are for independent noise. This larger, final result for $M_4^2(s-a)$ in correlated noise implies a larger $M_4^2(s-p)$ in correlated noise, which contains $M_4^2(s-a)$ as a subset. The larger *s-a* and *s-p* PDF second moments, or variances, translate into poorer detection. In summary, the correlated noise has a negligible effect on the *s-a* and *s-p* PDF means, but causes the *s-a* and *s-p* PDF variances to be larger than they would be for independent noise, ultimately leading to poorer detection in the correlated noise at a given SNR.

$n_1^2(t)$ and $n_2^2(t)$ are correlated in the positive sense if $n_1(t)$ and $n_2(t)$ are positively correlated or negatively correlated. It is expected that the noise on a hydrophone array between phones i and j will ordinarily be positively or negatively correlated, which will lead to correlation in the positive sense for $n_1^2(t)$ and $n_2^2(t)$.

The difference in detection results for independent and correlated noise for $m^x_{4(2,2)}$ does not occur when using $m^x_{4(3,1)}$. To understand this, consider the *s-p* PDF mean for $m^x_{4(3,1)}$ (Eq. (2d)), which is given by

$$M^q_4(s-p) = E\left\{ \Delta t \sum_{k=0}^{N-1} \begin{bmatrix} s^4(t) + n_1^3(t)\,n_2(t) \\ + 3s^3(t)\,n_1(t) + + 3s^2(t)\,n_1^2(t) + s(t)\,n_1^3(t) + s^3(t)\,n_2(t) \\ + 3s^2(t)\,n_1(t)\,n_2(t) + 3s(t)\,n_1^2(t)\,n_2(t) \end{bmatrix}^q \right\} \tag{B5}$$

with $q = 1$. As with the corresponding expression for $m^x_{4(2,2)}$ in Eq. (B1), terms 3, 4, 5, 6, 7, and 8 do not reflect differences in detection results for independent and correlated noise. The remaining term in this case is $\Delta t \sum_{k=0}^{N-1} n_1^3(t) n_2(t)$, which is the only term contributing to $M^q_4(s-a)$, and is small on average when $q = 1$. Now consider the second moment, or variance for $M^q_4(s-a)$, which is given by

$$M^2_4(s-a) = (\Delta t)^2 E\left\{ \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} n_1^3(t_1)\,n_1(t_2)\,n_2^3(t_1)\,n_2(t_2) \right\}. \tag{B6}$$

This expression does not contain any terms with two squared noise factors, as is the case with the $\Delta t \sum_{k=0}^{N-1} n_1^2(t) n_2^2(t)$ term in $m^x_{4(2,2)}$ and, thus, the $m^x_{4(3,1)}$ detector is not noticeably affected by the spatial correlation of the noise.

Appendix C

SUMMARY TABLES FOR DETECTION RESULTS AT $P_d = 0.5$

The detection curves shown in Figs. 7–24 are each interpolated at $P_d = 0.5$ to give the corresponding SNR in decibels. The SNRs are given in the first three columns of each table. The SNR gains are defined as the differences between the second order moment SNRs and the third and fourth order moment SNRs, and are given in the fourth and fifth columns of each table. Positive SNR gains imply that the higher order moment performs better. Differences between the cross correlation SNRs for the Gaussian noise with two or p channels are caused by the use of different realizations of noise in the simulations and are generally small.

Table C1 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in a Single Channel of Gaussian Noise

NOISE: GAUSSIAN		CHANNELS: 1		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–10.08	—	–9.64	—	–0.44
Pulse	–10.20	–12.47	–13.67	2.27	3.47
50-Hz Sinusoid	–12.37	—	–11.62	—	–0.75
49- to 51-Hz Sinusoid	–12.43	–3.01	–12.46	–9.42	0.03
Linear FM Sweep	–12.43	—	–11.68	—	–0.75
Nonlinear FM Sweep	–12.27	—	–11.30	—	–0.97

Table C2 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Two Channels of Gaussian Noise with $m^x_{4(2,2)}$

NOISE: GAUSSIAN		CHANNELS: 2		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–11.97	—	–10.36	—	–1.61
Pulse	–11.28	–14.65	–16.24	3.37	4.96
50-Hz Sinusoid	–13.68	—	–12.70	—	–0.98
49- to 51-Hz Sinusoid	–13.80	–5.20	–13.85	–8.60	0.05
Linear FM Sweep	–13.9	—	–12.48	—	–1.51
Nonlinear FM Sweep	–13.69	—	–12.67	—	–1.09

Table C3 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Two Channels of Gaussian Noise with $m^x_{4(3,1)}$

NOISE: GAUSSIAN		CHANNELS: 2		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	−11.92	—	−10.82	—	−1.10
Pulse	−12.36	−14.41	−15.89	2.05	3.53
50-Hz Sinusoid	−13.65	—	−13.12	—	−0.53
49- to 51-Hz Sinusoid	−13.86	−4.88	−13.90	−8.98	−0.04
Linear FM Sweep	−13.69	—	−13.00	—	−0.69
Nonlinear FM Sweep	−14.00	—	−13.13	—	−0.87

Table C4 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in p Channels of Gaussian Noise

NOISE: GAUSSIAN		CHANNELS: p		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	−11.97	—	−7.83	—	−4.14
Pulse	−12.40	−16.40	−18.37	4.00	5.97
50-Hz Sinusoid	−14.06	—	−11.04	—	−3.02
49- to 51-Hz Sinusoid	−13.43	−6.35	−13.77	−7.08	0.34
Linear FM Sweep	−13.91	—	−10.71	—	−3.20
Nonlinear FM Sweep	−14.17	—	−10.78	—	−3.39

Table C5 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in a Single Channel of Gaussian Noise with Prefiltering

NOISE: GAUSSIAN		CHANNELS: 1		PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	−17.72	—	−17.48	—	−0.24
Pulse	−12.46	−15.61	−16.86	3.15	4.40
50-Hz Sinusoid	−20.81	—	−21.18	—	0.37
49- to 51-Hz Sinusoid	−19.23	—	−20.54	—	1.31
Linear FM Sweep	−21.05	−12.38	−21.44	−8.67	0.39
Nonlinear FM Sweep	−16.10	—	−16.15	—	0.05

Table C6 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Two Channels of Gaussian Noise with Prefiltering and $m^x_{4(2,2)}$

NOISE: GAUSSIAN	CHANNELS: 2			PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–20.30	—	–19.45	—	–0.85
Pulse	–14.07	–17.72	–19.09	3.65	5.02
50-Hz Sinusoid	–23.10	—	–23.50	—	0.40
49- to 51-Hz Sinusoid	–21.08	—	–23.05	—	1.97
Linear FM Sweep	–22.91	–14.22	–22.92	–8.69	0.01
Nonlinear FM Sweep	–18.58	—	–17.90	—	–0.68

Table C7 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Two Channels of Gaussian Noise with Prefiltering and $m^x_{4(3,1)}$

NOISE: GAUSSIAN	CHANNELS: 2			PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–20.42	—	–19.97	—	–0.45
Pulse	–14.18	–17.98	–19.32	3.80	5.14
50-Hz Sinusoid	–22.53	—	–23.14	—	0.61
49- to 51-Hz Sinusoid	–21.30	—	–22.99	—	1.69
Linear FM Sweep	–23.26	–14.11	–23.70	–9.15	0.44
Nonlinear FM Sweep	–18.32	—	–18.29	—	0.06

Table C8 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in p Channels of Gaussian Noise with Prefiltering

NOISE: GAUSSIAN	CHANNELS: p			PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–19.86	—	20.01	—	0.15
Pulse	–14.35	–19.08	–21.89	4.73	7.54
50-Hz Sinusoid	–23.16	—	–24.82	—	1.66
49- to 51-Hz Sinusoid	–21.09	—	–24.97	—	3.88
Linear FM Sweep	–22.66	–16.30	–24.94	–6.36	2.28
Nonlinear FM Sweep	–17.63	—	–17.62	—	–0.01

Table C9 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channel 2 of the Shipping Noise

NOISE: SHIPPING		CHANNELS: 1		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	−2.28	—	−1.46	—	−0.82
Pulse	−2.26	−9.70	−8.73	7.44	6.47
50-Hz Sinusoid	−3.55	—	−3.19	—	−0.36
49- to 51-Hz Sinusoid	−3.31	−1.83	−4.89	−1.48	1.58
Linear FM Sweep	−3.40	—	−3.40	—	0.00
Nonlinear FM Sweep	−3.36	—	−3.41	—	0.05

Table C10 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channel 43 of the Shipping Noise

NOISE: SHIPPING		CHANNELS: 1		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	−3.46	—	−2.97	—	−0.49
Pulse	−3.61	−11.89	−9.53	8.28	5.92
50-Hz Sinusoid	−3.56	—	−3.75	—	0.19
49- to 51-Hz Sinusoid	−3.70	−2.02	−5.27	−1.68	1.57
Linear FM Sweep	−3.65	—	−3.77	—	0.12
Nonlinear FM Sweep	−3.59	—	−3.86	—	0.27

Table C11 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channels 2 and 43 of the Shipping Noise and $m^x_{4(2,2)}$

NOISE: SHIPPING		CHANNELS: 2		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	−8.39	—	−2.87	—	−5.52
Pulse	−8.39	−13.13	−11.43	4.74	3.04
50-Hz Sinusoid	−9.47	—	−4.94	—	−4.53
49- to 51-Hz Sinusoid	−9.68	−3.39	−7.09	−6.29	−2.59
Linear FM Sweep	−9.74	—	−5.09	—	−4.65
Nonlinear FM Sweep	−9.56	—	−5.00	—	−4.56

Table C12 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channels 2 and 43 of the Shipping Noise and $m^{x}_{4(3,1)}$

NOISE: SHIPPING		CHANNELS: 2		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–8.30	—	–6.59	—	–1.71
Pulse	–8.21	–12.93	–13.09	4.72	4.88
50-Hz Sinusoid	–9.23	—	–7.93	—	–1.30
49- to 51-Hz Sinusoid	–9.62	–3.20	–9.83	–6.42	0.21
Linear FM Sweep	–9.56	—	–8.37	—	–1.19
Nonlinear FM Sweep	–9.39	—	–8.03	—	–1.36

Table C13 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channels 2, 43, 16, and 31 of the Shipping Noise

NOISE: SHIPPING		CHANNELS: p		PREFILTERING: NO	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–7.93	—	–7.04	—	–0.89
Pulse	–7.70	–12.78	–16.43	5.08	8.73
50-Hz Sinusoid	–8.90	—	–9.94	—	1.04
49- to 51-Hz Sinusoid	–9.10	–4.63	–12.42	–4.47	3.32
Linear FM Sweep	–9.10	3.68	–9.93	–12.78	0.83
Nonlinear FM Sweep	–9.14	—	–9.68	—	0.54

Table C14 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channel 2 of the Shipping Noise with Prefiltering

NOISE: SHIPPING		CHANNELS: 1		PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–13.99	—	–13.53	—	–0.46
Pulse	–2.82	–9.60	–8.85	6.78	6.03
50-Hz Sinusoid	–9.04	—	–9.00	–3.72	–0.04
49- to 51-Hz Sinusoid	–7.83	—	–10.52	—	2.69
Linear FM Sweep	–8.28	–2.21	–9.21	–6.07	0.93
Nonlinear FM Sweep	–5.05	3.68	–5.53	–8.73	0.48

Table C15 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channel 43 of the Shipping Noise with Prefiltering

NOISE: SHIPPING		CHANNELS: 1		PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–10.26	—	–9.40	—	–0.86
Pulse	–4.48	–12.42	–10.33	7.94	5.85
50-Hz Sinusoid	–8.66	–2.84	–9.89	–5.82	1.23
49- to 51-Hz Sinusoid	–7.62	—	–10.11	—	2.49
Linear FM Sweep	–7.81	–2.31	–8.47	–5.50	0.66
Nonlinear FM Sweep	–5.33	—	–6.25	—	0.92

Table C16 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channels 2 and 43 of the Shipping Noise with Prefiltering and $m^x_{4(2,2)}$

NOISE: SHIPPING		CHANNELS: 2		PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–13.45	—	–12.90	—	–0.5527
Pulse	–8.58	–13.05	–11.94	4.47	3.35
50-Hz Sinusoid	–13.42	—	–10.94	—	–2.49
49- to 51-Hz Sinusoid	–12.24	—	–12.31	—	0.0639
Linear FM Sweep	–12.76	–2.24	–10.47	–10.52	–2.29
Nonlinear FM Sweep	–9.97	3.87	–6.66	–13.85	–3.32

Table C17 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channels 2 and 43 of the Shipping Noise with Prefiltering and $m^x_{4(3,1)}$

NOISE: SHIPPING		CHANNELS: 2		PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	–13.31	—	–13.24	—	–0.06
Pulse	–8.43	–12.87	–13.02	4.44	4.59
50-Hz Sinusoid	–13.34	—	–14.30	—	0.95
49- to 51-Hz Sinusoid	–12.03	—	–14.39	—	2.36
Linear FM Sweep	–12.65	–2.07	–13.26	–10.58	0.61
Nonlinear FM Sweep	–9.87	3.87	–10.13	–13.74	0.26

Table C18 — SNR in Decibels at $P_d = 0.5$ and SNR Gains Over the Second Moment Detector for Transients in Channels 2, 43, 16, and 31 of the Shipping Noise with Prefiltering

NOISE: SHIPPING		CHANNELS: p		PREFILTERING: YES	
SIGNAL	SECOND MOMENT	THIRD MOMENT	FOURTH MOMENT	THIRD MOMENT GAIN	FOURTH MOMENT GAIN
Whale	−13.05	—	−13.48	—	0.43
Pulse	−8.02	−13.03	−16.70	5.01	8.68
50-Hz Sinusoid	−12.96	—	−15.49	—	2.53
49- to 51-Hz Sinusoid	−11.59	—	−16.12	—	4.52
Linear FM Sweep	−12.11	−5.09	−14.37	−7.02	2.26
Nonlinear FM Sweep	−9.55	3.16	−11.42	−12.70	1.88